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# TO THE PROBLEM OF BOUNDORY CONDITIONS OF NULL STRING

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This paper analyzes the Einstein equations for a closed null string, collapsing in a plane $z = 0$. It is shown that the solution of Minkowski can't be regarded as the asymptotic behavior of the gravitational field generated by a null string.
***Keywords:*** null string, a scalar field, boundary conditions, Minkowski space.

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## INTRODUCTION

Historically, the problem of boundary conditions in the general theory of relativity was related to questions of cosmology and therefore formulated as the problem of the boundary conditions at infinity. In [1], Einstein outlined three possible solutions:

1. At spatial infinity, an appropriate choice of the coordinate system to the metric tends to the metric of flat Minkowski space;
2. There is no boundary conditions, which could claim to universal validity (each task should be an individual decision of this question);
3. General equations of the field to be changed by the introduction of additional (cosmological) term so that the space was closed, than remove the question of boundary conditions.

Although in the same paper, it was noted that the hypothesis 1 is not always consistent with the notion of the relativity of inertia and does not agree with some statistical considerations, Assumption 2 does not actually correspond to any solution of the problem, and hence the rejection of its solutions, and in the case of the 3 we get a generalization of the field, which is not supported by our actual knowledge of gravitation.

In [2], based on the classification of gravitational fields on the algebraic structure of the curvature tensor invariant formulated solution of the boundary conditions.

First of all, it should be noted that, according to [2], type of space $V_n$, for a given $n$, determined by the so-called characteristic $\lambda$ – matrix $\left(R_{\alpha\beta} - \lambda g_{\alpha\beta}\right)$, where $R_{\alpha\beta}$ – tensor Richie, $g_{\alpha\beta}$ – the metric tensor of the space $V_n, \alpha, \beta = 1,...,n$. When $n = 4$ there are only three types of gravitational fields of general form $T_i$, $i = 1,2,3$, in terms of the algebraic structure of the tensor space-matter. These types correspond to the three possible types of characteristics $\lambda$ – matrix.

Formulated in [2], the principle of superposition states that the boundary conditions: if there is a gravitational field $T_i$ $(i = 1,2,3)$, then for areas where $T_{\alpha\beta} \to 0$, and, accordingly, $R_{\alpha\beta} \to 0$, ($T_{\alpha\beta}$ – energy-momentum tensor) the components of the metric tensor $g_{\alpha\beta}$ of the gravitational field should be arbitrarily close to the corresponding components of the metric tensor space, which is the space of the same type as the given, and allows the maximum possible for this type of space group of motions.

For example, for spaces $T_1$, curvature, which is expressed in the deviation from zero of the curvature tensor, is caused, by the presence of gravitational mass. As the distance from the masses (for example, on a hypersphere of infinite radius to the Schwarzschild solution, in polar coordinates) of the gravitational action is weakened, “curvature” of space is smoothed, it becomes more uniform and tends to space $T_1$ as possible for this type of space group movements (Minkowski space).

Proposed in [2] formulation solution of the boundary conditions is a group-invariant and independent of the choice of coordinates. It is important to note that an attempt to put the solution of a problem for the space of a specified type of boundary conditions of another type should lead to a contradiction, since the degeneracy of the metric and the change in the type of space in this case is not physically be motivated.

In other words, if for a group of motions of spaces $T_1$ is a group $G_{10}$, for the space $T_2$ group $G_6$, and for the space $T_3$ group $G_2$, then flat Minkowski space can not be considered as the boundary conditions for the space type $T_2$ and $T_3$.

Space generated by the different distribution of real massless scalar field, the special cases are different and the “string-like” distribution, refer to spaces of the type $T_3$.

The goal was to show that the gravitational field generated by a closed “thick” null string collapsing in a plane $z = 0$ can’t be as asymptotic flat Minkowski solution. In a cylindrical coordinate system $x^0 = t$, $x^1 = \rho$, $x^2 = \theta$, $x^3 = z$, functions $x^m(\tau,\sigma)$, determine the trajectory of the closed null string, have the form:

$$t = \tau, \rho = -\tau, \theta = \sigma, z = 0, \tau \in (-\infty;0], \tag{1}$$

where $\tau$ and $\sigma$ – there are options on the world surface null string.

Since the zero-string implement zero tension limit of string theory [3], the components of energy-momentum tensor for the null strings are of the form

$$T^{mn}\sqrt{-g} = \gamma \int d\tau d\sigma x^m_{,\tau} x^n_{,\tau} \delta^4\left(x^l - x^l(\tau,\sigma)\right), \tag{2}$$

where the indices $m,n,l$ take values 0,1,2,3, $x^m_{,\tau} = \partial x / \partial \tau$, $g = |g_{mn}|$, $g_{mn}$ – the metric tensor of the outer space, $\gamma = const$. For (1) non-zero components are such energy-momentum tensor (2)

$$T^{00} = T^{11} = -T^{01} = \frac{\gamma}{\sqrt{-g}} \delta(z)\delta(\eta), \tag{3}$$

where $\eta = t + \rho$.

Using the results of [4, 5], the general expression of the quadratic form describing the movement of a null string, determines the trajectory of (1) can be written as

$$ds^2 = e^{2\nu}(dt)^2 - A(d\rho)^2 - B(d\theta)^2 - e^{2\mu}(dz)^2, \tag{4}$$

where: $\nu, \mu, A, B$ are functions of variables $t, \rho, \theta, z$. At this stage it is important to note that the decision of the Minkowski one of the special cases (4)

$$e^{2\nu} = e^{2\mu} = A = 1,\ B = \rho^2.$$

Since the trajectory of (1) must be one of the solutions of the equations of motion null string, it is possible to obtain restrictions on the metric functions, in which the trajectory of the zero-string remains unchanged. Motion null strings in a pseudo-space-time is determined by the following system of equations [2]:

$$x^m_{,\tau\tau} + \Gamma^m_{pq} x^p_{,\tau} x^q_{,\tau} = 0, \tag{5}$$

$$g_{mn} x^m_{,\tau} x^n_{,\tau} = 0,\ g_{mn} x^m_{,\tau} x^n_{,\sigma} = 0, \tag{6}$$

where (5) – this is the equation of motion and the null string, (6) – constraint equation, $\Gamma^m_{pq}$ – Christoffel symbols of the external space-time. The first equation of (6) to (1) has the form $e^{2\nu} - A = 0$, wherefrom

$$e^{2\nu} = A. \tag{7}$$

The remaining equations in (5) and (6) to (4) subject to (7) are reduced to a single equation $\nu_{,t} - \nu_{,\rho} = 0$, where from

$$\nu = \nu(\eta, \theta, z). \tag{8}$$

Analysis of the Einstein equations for the quadratic form (4) and a component of the energy-momentum (4), under the conditions (9), (10), can complete the definition of the functional dependence of metric functions, namely:

$$\mu = \mu(\eta, \theta, z),\ B = B(\eta, \theta, z), \tag{9}$$

while the system itself is the Einstein equations can be written as

$$-\mu_{,\eta\eta} - \frac{1}{2}\frac{B_{,\eta\eta}}{B} - (\mu_{,\eta})^2 + \frac{1}{4}\left(\frac{B_{,\eta}}{B}\right)^2 + 2\nu_{,\eta}\left(\mu_{,\eta} + \frac{1}{2}\frac{B_{,\eta}}{B}\right) = \chi T_{00}, \tag{10}$$

$$\begin{gathered} e^{2(\nu-\mu)}\left(\nu_{,zz} + (\nu_{,z})^2 + \frac{1}{2}\frac{B_{,zz}}{B} - \frac{1}{4}\left(\frac{B_{,z}}{B}\right)^2 - \mu_{,z}\nu_{,z} - \frac{1}{2}\frac{B_{,z}}{B}(\mu_{,z} - \nu_{,z})\right) + \\ + \frac{e^{2\nu}}{B}\left(\nu_{,\theta\theta} + \mu_{,\theta\theta} + (\nu_{,\theta})^2 + (\mu_{,\theta})^2 - \frac{1}{2}\frac{B_{,\theta}}{B}(\nu_{,\theta} + \mu_{,\theta}) + \nu_{,\theta}\mu_{,\theta}\right) = 0 \end{gathered}, \tag{11}$$

$$\frac{B_{,\eta z}}{B} + 2\nu_{,\eta z} - \frac{1}{2}\frac{B_{,\eta}}{B}\frac{B_{,z}}{B} - \frac{B_{,\eta}}{B}\nu_{,z} - \frac{B_{,z}}{B}\mu_{,\eta} - 2\mu_{,\eta}\nu_{,z} = 0, \tag{12}$$

$$\frac{B}{e^{2\mu}}\left(2\nu_{,zz} + 3(\nu_{,z})^2 - 2\nu_{,z}\mu_{,z}\right) - 2\nu_{,\theta}\mu_{,\theta} - (\nu_{,\theta})^2 = 0, \tag{13}$$

$$\left(\nu_{,z}\right)^2+\nu_{,z}\frac{B_{,z}}{B}-\frac{e^{2\mu}}{B}\left(2\nu_{,\theta\theta}+3\left(\nu_{,\theta}\right)^2+\nu_{,\theta}\frac{B_{,\theta}}{B}\right)=0\,, \tag{14}$$

$$\nu_{,\eta\theta}+\mu_{,\eta\theta}-\frac{1}{2}\frac{B_{,\eta}}{B}\left(\nu_{,\theta}+\mu_{,\theta}\right)-\mu_{,\eta}\left(\nu_{,\theta}-\mu_{,\theta}\right)=0\,, \tag{15}$$

$$\nu_{,\theta z}+\nu_{,z}\left(\nu_{,\theta}-\mu_{,\theta}\right)-\nu_{,\theta}\frac{B_{,z}}{B}=0\,. \tag{16}$$

As follows from (3) is a string, all the components of the string energy-momentum tensor are zero and non-zero (to infinity) directly on the string, which allows us to study the system of Einstein's equations for the problem in two ways:

- Limit the analysis to the “external” problems.
- Of the component string energy-momentum tensor as the limit of a “thick” distribution and analysis of Einstein's equations for this “thick” distribution.

As shown in [6-8] analysis of “external” problem leads to a large number of vacuum solutions of Einstein's equations satisfying the symmetries of the problem, however, remains unclear criteria allowing you to select from this set of solutions describe the gravitational field of a null string moving along the path (1). When you try to consider the components of the energy-momentum tensor of the string as the limit of a “thick” distribution, for example, a simple replacement of the delta functions in the tensor (2) the appropriate delta-function sequences are possible inaccuracies related to the fact that it is unclear how to account for the possible emergence of terms (factors) that tend to zero (constant) the contraction of “thick” distribution of a one-dimensional object. Therefore, it is easier initially considered a “well-defined” “thick” distribution, for example, a real massless scalar field (because the task at hand, we consider the scalar null object), and then pull it to the string configuration required, while requiring that the components of the energy momentum of a scalar field in the limit of this compression asymptotically coincide with the components of the tensor (3).

For (7), (8) the expression (4) takes the form

$$ds^2=e^{2\nu}\left(\left(dt\right)^2-\left(d\rho\right)^2\right)-B\left(d\theta\right)^2-e^{2\mu}\left(dz\right)^2\,. \tag{17}$$

**ANALYSIS OF THE RESULTING SYSTEM OF EINSTEIN FOR THICK DISTRIBUTION**

Energy-momentum tensor for a real massless scalar field is [7]

$$T_{\alpha\beta}=\varphi_{,\alpha}\varphi_{,\beta}-\frac{1}{2}g_{\alpha\beta}L\,, \tag{18}$$

where $L=g^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta}$, $\varphi_{,\alpha}=\partial\varphi/\partial x^{\alpha}$, $\varphi$ – potential of the scalar field, indices $\alpha,\beta$ range 0,1,2,3. For the self-consistency of the Einstein equations will require

$$T_{\alpha\beta}=T_{\alpha\beta}\left(\eta,\theta,z\right)\rightarrow\varphi=\varphi\left(\eta,\theta,z\right). \tag{19}$$

Writing equation (18),

$$T_{00}=\left(\varphi_{,\eta}\right)^2+\frac{e^{2\nu}}{2}\left(\frac{\left(\varphi_{,z}\right)^2}{e^{2\mu}}+\frac{\left(\varphi_{,\theta}\right)^2}{B}\right),\ T_{11}=\left(\varphi_{,\eta}\right)^2-\frac{e^{2\nu}}{2}\left(\frac{\left(\varphi_{,z}\right)^2}{e^{2\mu}}+\frac{\left(\varphi_{,\theta}\right)^2}{B}\right),\ T_{01}=\left(\varphi_{,\eta}\right)^2$$

$$T_{22}=\frac{1}{2}\left(\left(\varphi_{,\theta}\right)^2-\frac{B}{e^{2\mu}}\left(\varphi_{,z}\right)^2\right),\ T_{03}=T_{13}=\varphi_{,\eta}\varphi_{,z},\ T_{12}=\varphi_{,\eta}\varphi_{,\theta},\tag{20}$$

$$T_{23}=\varphi_{,\theta}\varphi_{,z},\ \ T_{33}=\frac{1}{2}\left(\left(\varphi_{,z}\right)^2-\frac{e^{2\mu}}{B}\left(\varphi_{,\theta}\right)^2\right).$$

For (17) and (20) the system of Einstein's equations can be written as

$$-\mu_{,\eta\eta}-\frac{1}{2}\frac{B_{,\eta\eta}}{B}-\left(\mu_{,\eta}\right)^2+\frac{1}{4}\left(\frac{B_{,\eta}}{B}\right)^2+2\nu_{,\eta}\left(\mu_{,\eta}+\frac{1}{2}\frac{B_{,\eta}}{B}\right)=\chi\left(\varphi_{,\eta}\right)^2,\tag{21}$$

$$e^{2(\nu-\mu)}\left(\nu_{,zz}+\left(\nu_{,z}\right)^2+\frac{1}{2}\frac{B_{,zz}}{B}-\frac{1}{4}\left(\frac{B_{,z}}{B}\right)^2-\mu_{,z}\nu_{,z}-\frac{1}{2}\frac{B_{,z}}{B}\left(\mu_{,z}-\nu_{,z}\right)\right)+$$

$$+\frac{e^{2\nu}}{B}\left(\nu_{,\theta\theta}+\mu_{,\theta\theta}+\left(\nu_{,\theta}\right)^2+\left(\mu_{,\theta}\right)^2-\frac{1}{2}\frac{B_{,\theta}}{B}\left(\nu_{,\theta}+\mu_{,\theta}\right)+\nu_{,\theta}\mu_{,\theta}\right)=\qquad,\tag{22}$$

$$=-\frac{\chi e^{2\nu}}{2}\left(\frac{\left(\varphi_{,z}\right)^2}{e^{2\mu}}+\frac{\left(\varphi_{,\theta}\right)^2}{B}\right)$$

$$\frac{1}{2}\frac{B_{,\eta z}}{B}+\nu_{,\eta z}-\frac{1}{4}\frac{B_{,\eta}}{B}\frac{B_{,z}}{B}-\frac{1}{2}\frac{B_{,\eta}}{B}\nu_{,z}-\frac{1}{2}\frac{B_{,z}}{B}\mu_{,\eta}-\mu_{,\eta}\nu_{,z}=-\chi\varphi_{,\eta}\varphi_{,z},\tag{23}$$

$$\frac{B}{e^{2\mu}}\left(2\nu_{,zz}+3\left(\nu_{,z}\right)^2-2\nu_{,z}\mu_{,z}\right)-2\nu_{,\theta}\mu_{,\theta}-\left(\nu_{,\theta}\right)^2=\frac{\chi}{2}\left(\left(\varphi_{,\theta}\right)^2-\frac{B}{e^{2\mu}}\left(\varphi_{,z}\right)^2\right),\tag{24}$$

$$\left(\nu_{,z}\right)^2+\nu_{,z}\frac{B_{,z}}{B}-\frac{e^{2\mu}}{B}\left(2\nu_{,\theta\theta}+3\left(\nu_{,\theta}\right)^2+\nu_{,\theta}\frac{B_{,\theta}}{B}\right)=\frac{\chi}{2}\left(\left(\varphi_{,z}\right)^2-\frac{e^{2\mu}}{B}\left(\varphi_{,\theta}\right)^2\right),\tag{25}$$

$$\nu_{,\eta\theta}+\mu_{,\eta\theta}-\frac{1}{2}\frac{B_{,\eta}}{B}\left(\nu_{,\theta}+\mu_{,\theta}\right)-\mu_{,\eta}\left(\nu_{,\theta}-\mu_{,\theta}\right)=\chi\varphi_{,\eta}\varphi_{,\theta},\tag{26}$$

$$\nu_{,\theta z}+\nu_{,z}\left(\nu_{,\theta}-\mu_{,\theta}\right)-\nu_{,\theta}\frac{B_{,z}}{B}=\chi\varphi_{,\theta}\varphi_{,z}.\tag{27}$$

Since the covariant derivative of the components of the Einstein tensor is zero, i.e. $G^{\beta}_{\alpha;\beta}=0$, where $G^{\beta}_{\alpha}$ – the Einstein tensor, the semicolon denotes the covariant derivative, demanding the equality

$$T^{\beta}_{\alpha;\beta}=0,$$

for (18), we obtain the equation that must satisfy scalar field potential [7]

$$\left(g^{\alpha\beta}\varphi_{,\alpha}\right)_{;\beta}=0.\tag{28}$$

For (17), equation (28) takes the form

$$\frac{\varphi_{,z}}{e^{2\mu}}\frac{\partial}{\partial z}\left(\ln\left(\varphi_{,z}e^{2\nu}e^{-\mu}\sqrt{B}\right)\right)-\frac{\varphi_{,\theta}}{B}\frac{\partial}{\partial\theta}\left(\ln\left(\varphi_{,\theta}e^{2\nu}e^{-\mu}\sqrt{B}\right)\right)=0. \tag{29}$$

Total (24) and (25) is

$$\frac{\nu_{,z}}{e^{2\mu}}\frac{\partial}{\partial z}\left(\ln\left(\nu_{,z}e^{2\nu}e^{-\mu}\sqrt{B}\right)\right)-\frac{\nu_{,\theta}}{B}\frac{\partial}{\partial\theta}\left(\ln\left(\nu_{,\theta}e^{2\nu}e^{-\mu}\sqrt{B}\right)\right)=0. \tag{30}$$

Comparing (29) and (30) that

$$\nu_{,z}=c(\eta)\varphi_{,z} \text{ и } \nu_{,\theta}=c(\eta)\varphi_{,\theta}. \tag{31}$$

Integrating (31) we

$$\nu(\eta,\theta,z)=c(\eta)\varphi(\eta,\theta,z)+\omega(\eta,\theta) \tag{32}$$

and

$$\nu(\eta,\theta,z)=c(\eta)\varphi(\eta,\theta,z)+\omega(\eta,z), \tag{33}$$

It follows that

$$\nu(\eta,\theta,z)=c(\eta)\varphi(\eta,\theta,z)+\omega_0(\eta). \tag{34}$$

Consider the resulting system of equations (21) – (27) for the distribution of the scalar field, concentrated within the “ton” of the ring, for which the variables $\eta$ and $z$ change within

$$\eta\in[-\Delta\eta;\Delta\eta],\ z\in[-\Delta z;\Delta z], \tag{35}$$

where the positive constants $\Delta\eta$ and $\Delta z$ define the “thickness” of the ring

$$\Delta\eta<<1,\ \Delta z<<1, \tag{36}$$

and in the limit of compression of the “thin” rings in a one-dimensional object (a null string)

$$\Delta\eta\to0,\ \Delta z\to0. \tag{37}$$

Then the space-time in which moves a “smear” null string, and for which the variables $\eta$ and $z$ change within

$$\eta\in(-\infty;+\infty),\ z\in(-\infty;+\infty),\ \theta\in[0;2\pi] \tag{38}$$

can be divided into three areas:

- region I, for which

$$\eta\in(-\infty;-\Delta\eta)\cup(\Delta\eta;+\infty),\ z\in(-\infty;+\infty),\ \theta\in[0;2\pi], \tag{39}$$

- region II, for which

$$\eta\in[-\Delta\eta;\Delta\eta],\ z\in(-\infty;-\Delta z)\cup(\Delta z;+\infty),\ \theta\in[0;2\pi], \tag{40}$$

- region III, for which

$$z\in[-\Delta z;\Delta z],\ \eta\in[-\Delta\eta;\Delta\eta],\ \theta\in[0;2\pi]. \tag{41}$$

Since the contraction of the scalar field in the string equations (21) – (27) for the scalar field must asymptotically tend to the system (10) – (16) for a closed null string, in the region I, II

$$\varphi\to0,\ \varphi_{,z}\to0,\ \varphi_{,\eta}\to0,\ \varphi_{,\theta}\to0 \tag{42}$$

and in the region III, in general,

$$\varphi \neq 0\,,\ \varphi_{,z} \neq 0\,,\ \varphi_{,\eta} \neq 0\,. \tag{43}$$

Comparing equations (10) – (16) for a closed null string with the system (21) – (27), it can be concluded that the contraction of the scalar field in the string, that is, when $\Delta\eta \to 0$, $\Delta z \to 0$

$$\left(\varphi_{,\eta}\right)^2 \to \infty\,,\left(\frac{\left(\varphi_{,z}\right)^2}{e^{2\mu}} + \frac{\left(\varphi_{,\theta}\right)^2}{B}\right) \to 0\,,$$

$$\left(\left(\varphi_{,\theta}\right)^2 - \frac{B}{e^{2\mu}}\left(\varphi_{,z}\right)^2\right) \to 0\,,\ \varphi_{,\eta}\varphi_{,\theta} \to 0\,, \tag{44}$$

$$\left(\varphi_{,z}\varphi_{,\eta}\right) \to 0\,.$$

In the region I, according to (43), for any fixed value of the variable $\eta = \eta_0 \in (-\infty;-\Delta\eta) \cup (\Delta\eta;+\infty)$ for all values $z \in (-\infty;+\infty)$, the potential of the scalar field

$$\varphi(\eta_0, \theta, z) \to 0\,. \tag{45}$$

If we consider the distribution of the potential of a scalar field for each fixed variable $\eta = \eta_0 \in [-\Delta z; \Delta z]$, (region II and III), in the case where the variable $z \in (-\infty;-\Delta z) \cup (\Delta z;+\infty)$ (region II), must be made

$$\varphi(\eta_0, \theta, z) \to 0\,, \tag{46}$$

and for $z \in [-\Delta z; \Delta z]$ (region III)

$$\varphi(\eta_0, \theta, z) \neq 0\,. \tag{47}$$

For conditions (45) – (47) the potential distribution of a scalar field is conveniently written as

$$\varphi(z, \theta, \eta) = -\ln\left(\alpha(\eta, \theta) + \lambda(\eta, \theta) f(z, \theta)\right), \tag{48}$$

and the potential of the scalar field must satisfy closed null string, i.e.

$$\varphi(\eta, \theta, z)_{\theta=0} = \varphi(\eta, \theta, z)_{\theta=2\pi}\,. \tag{49}$$

Functions $\alpha(\eta,\theta)$ and $\lambda(\eta,\theta)$ symmetric with respect to inversion $\eta$ to $-\eta$:

$$\alpha(\eta, \theta) = \alpha(-\eta, \theta),\ \lambda(\eta, \theta) = \lambda(-\eta, \theta). \tag{50}$$

According to (49)

$$\alpha(\eta, \theta)_{\theta=0} = \alpha(\eta, \theta)_{\theta=2\pi}\,,\ \lambda(\eta, \theta)_{\theta=0} = \lambda(\eta, \theta)_{\theta=2\pi}\,,\ f(\theta, z)_{\theta=0} = f(\theta, z)_{\theta=2\pi}\,. \tag{51}$$

Function $\alpha(\eta,\theta) + \lambda(\eta,\theta) f(z,\theta)$ is bounded, i.e.

$$0 < \alpha(\eta, \theta) + \lambda(\eta, \theta) f(z, \theta) \leq 1\,, \tag{52}$$

and the potential of the scalar field (48), in (52), can range from

$$\varphi \to 0,\ \text{at}\ \alpha(\eta, \theta) + \lambda(\eta, \theta) f(z, \theta) = 1\,, \tag{53}$$

to

$$\varphi \to \infty,\ \text{at}\ \alpha(\eta, \theta) + \lambda(\eta, \theta) f(z, \theta) \to 0\,, \tag{54}$$

and in the region I, in accordance with (45) and (53)

$$\alpha(\eta,\theta)\to 1,\ \lambda(\eta,\theta)\to 0\,. \tag{55}$$

Since, by (48), the potential of the scalar field in the region II is zero, and for $\eta\in[-\Delta\eta;\Delta\eta]$ any fixed value of the variable $z=z_0\in(-\infty;-\Delta z)\cup(\Delta z;+\infty)$ must be done

$$\alpha(\eta,\theta)+\lambda(\eta,\theta)f(z_0,\theta)\to 1. \tag{56}$$

In the region III, $\varphi\neq 0$, so for the same values $\eta\in[-\Delta\eta;\Delta\eta]$ and $z=z_0\in[-\Delta z;\Delta z]$

$$0<\alpha(\eta,\theta)+\lambda(\eta,\theta)f(z_0,\theta)<1. \tag{57}$$

For (56) and for $z\in(-\infty;-\Delta z)\cup(\Delta z;+\infty)$

$$f(z,\theta)\to f_0=const\,, \tag{58}$$

moreover $f_0\neq 0$, and the functions $\alpha(\eta,\theta)$ and $\lambda(\eta,\theta)$ are linked

$$\lambda(\eta,\theta)=(1-\alpha(\eta,\theta))/\,f_0\,. \tag{59}$$

Substituting (58) and (59) to (56) that in the III ($\varphi\neq 0$)

$$0<\alpha(\eta,\theta)+(1-\alpha(\eta,\theta))f(z,\theta)/\,f_0<1, \tag{60}$$

then (54), (60) it follows that $\varphi\to\infty$

$$\alpha(\eta,\theta)\to 0\,,\ f(z,\theta)\to 0\,. \tag{61}$$

Thus, in the expression for the potential of the scalar field (48), and the limited functions $\alpha(\eta,\theta)$ and $f(z,\theta)$ for all $z\in(-\infty;+\infty)$ и $\eta\in(-\infty;+\infty)$ take values

$$0<\alpha(\eta,\theta)<1,\ 0<f(z,\theta)<f_0. \tag{62}$$

Behavior of the function $f(z,\theta)$ at $z\in(-\infty;-\Delta z)\cup(\Delta z;+\infty)$ defined in (58), and at $z\to 0$, according to (61)

$$f(z,\theta)\to 0\,. \tag{63}$$

Since the zero-string moving along the path (1) at any given time is a circle, then the potential of the scalar field (48) does not depend on $\theta$, i.e.

$$\varphi=\varphi(\eta,z). \tag{64}$$

For (60), equality (34) takes the form

$$\nu(\eta,z)=c(\eta)\varphi(\eta,z)+\omega(\eta), \tag{65}$$

taking into account (7) we find that

$$A=A(\eta,z). \tag{66}$$

For (64), (65) and (66) of (27) takes the form

$$\nu_{,z}\mu_{,\theta}=0\,, \tag{67}$$

it follows that

$$\mu=\mu(\eta,z). \tag{68}$$

Rewriting (25) to (64), (65) we find that

$$\left(\nu_{,z}\right)^2 + \nu_{,z}\frac{B_{,z}}{B} = \frac{\chi}{2}\left(\varphi_{,z}\right). \qquad (69)$$

As in (69) $\varphi$ and $\nu$ are functions of variables $\eta$ and $z$, and that $B_{,z}/B$ is a function only of variables $\eta$ and $z$, and where

$$B(\eta,\theta,z) = (\beta(\theta))^2\,\tilde{B}(\eta,z). \qquad (70)$$

Since the function $B$ in the quadratic form (15) is at $(d\theta)^2$, than you can always make a transformation of the coordinate system $d\theta' = \beta(\eta)d\theta$ in which the dependence $B$ on the variable $\theta$ is removed, so in the future without loss of generality we assume that

$$B = B(\eta,z). \qquad (71)$$

In [5] analyzed the Einstein equations for the metric functions are independent of the variable $\theta$ where it was shown that the only possible solution for the closed null string collapsing in the plane $z=0$ is

$$e^{2\nu(z,\eta)} = \frac{c_1(\eta)}{c_0}\left(\left(\int c_1(\eta)d\eta\right)^2\right)^{c(c-1)} \exp\{2\tilde{c}_2\varphi(z,\eta)\}, \qquad (72)$$

$$B(z,\eta) = \left(\left(\int c_1(\eta)d\eta\right)^2\right)^{1-c} \exp\{\tilde{c}_3\varphi(z,\eta)\}, \qquad (73)$$

$$e^{2\mu(z,\eta)} = \frac{1}{c_0^2}\left(\left(\int c_1(\eta)d\eta\right)^2\right)^{(1-c)(1-2c)} \left(\varphi_{,\eta}\right)^2 \exp\{(\tilde{c}_3 + 4\tilde{c}_2)\varphi(z,\eta)\}, \qquad (74)$$

From the above solutions can be seen that it is not under any values of the variables and the specified function does not lead to a solution of Minkowski.

**CONCLUSIONS**

This paper analyzes the Einstein equations for a closed null string collapsing in the plane $z=0$. Noted that the decision of Minkowski is a private case of the initial quadratic. However, it is shown that the solution of Minkowski can not be considered as an asymptotic behavior of the gravitational field generated by a null string. This result confirms the generally formulated in [2] invariant (group) solution of the boundary conditions in general relativity, based on the classification of gravitational fields on the algebraic structure of the curvature tensor.

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**Леляков О. П. До питання про граничних умовах нуль-струни / О. П. Леляков, А. С. Карпенко //** Вчені записки Таврійського національного університету імені В. И. Вернадського. Серія : Фізико-математичні науки. – 2013. – Т. 26 (65), № 2. – С. 79-88.
У даній роботі проведено аналіз системи рівнянь Ейнштейна для замкнутої нуль-струни, оj колапсуює в площині $z = 0$. Показано, що, рішення Мінковського не може розглядатися в якості асимптотики для гравітаційного поля породжуваного нуль-струною.
***Ключові слова:*** нуль-струна, скалярне поле, граничні умови, простір Мінковського.

**Леляков А. П. К вопросу о граничных условиях нуль-струны / А. П. Леляков, А. С. Карпенко //** Ученые записки Таврического национального университета имени В. И. Вернадского. Серия : Физико-математические науки. – 2013. – Т. 26 (65), № 2. – С. 79-88.
В данной работе проведен анализ системы уравнений Эйнштейна для замкнутой нуль-струны, коллапсирующей в плоскости $z = 0$. Показано, что, решение Минковского не может рассматриваться в качестве асимптотики для гравитационного поля порождаемого нуль-струной.
***Ключевые слова:*** нуль-струна, скалярное поле, граничные условия, пространство Минковского.

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