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INFLUENCE OF THE MAGNETIC ANISOTROPY ON THE FORMATION OF SPIN NEMATIC PROPERTIES

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In this work the model of anisotropic spin-1 non-Heisenberg magnet with exchange anisotropy is studied. It is shown that the anisotropic spin nematic state is realized in the system. The conditions of this phase stability and angle of vector-director orientation are determined. The dispersion laws for magnons of different polarization are calculated in anisotropic spin nematic phase.
Keywords: exchange anisotropy, anisotropic spin nematic, dispersion law.

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INTRODUCTION

It is well known that the complete description of the spin-1 systems requires account of both, the dipole and the quadrupolar magnetic moments. The average values of the spin per site is $\mathbf{m} = <\mathbf{S}>$, and quadrupolar spin average is $Q_{ij} =< S_i S_j + S_j S_i >$. The appearance of nontrivial values of these magnitudes lead to different ordered states. The magnetic state (ferromagnetic, antiferromagnetic) is characterized by nontrivial value of the average spin $\mathbf{m}$, but for the nematic state $\mathbf{m} = 0$, and the spontaneous symmetry breaking is manifested in nontrivial properties of quadrupole average values Q_{ij}. The nematic state usually appears in isotropic model with bilinear and biquadratic interaction of the spins. However, the isotropic model of a magnetic is idealized; therefore, as to approach the real systems we have to take into account the anisotropy. Account of the magnetic anisotropy leads to appearing different magnetic phases, for example, the easy-axis or the easy-plane ones in ferromagnetics. The magnetic anisotropy can be introduced by two ways: either considering anisotropic properties of magnetic ion (single-ion anisotropy), or introducing anisotropy of exchange interaction of the spins. Usually, if the anisotropy is small enough, then both kinds of anisotropy lead to the similar effects for ferromagnetic state, namely, the easy-axis state, or the easy-plane state is realized in the ferromagnet. Their properties can be studied within the frames of Landau-Lifshitz equation. It will be shown below that the single-ion anisotropy and the exchange anisotropy influence the properties formation in the nematic state differently, apart from the ferromagnetic one.

The spin nematic dynamics is described by the vector-director, which direction is infinitely degenerated in the isotropic case. So, the isotropic spin nematic spectrum contains two modes, degenerated on polarization, which soften in the transition point into the ferromagnetic state [2]. The magnons in isotropic spin nematic have all properties of

Goldstown's excitations far from the transition point. In the limit of small wave vector they have a linear dispersion law and damping is quadratic in frequency [3].

Account of the single-ion anisotropy partially removes the degeneracy of the ground state. So, the anisotropic spin nematic state with the anisotropy easy-plan type for vector-director can be realized in the system [4]. The result of such degeneration is the appearance of the activation mode in the anisotropic spin nematic spectrum. This situation has been studied in details in work [4], where the dispersion laws and damping are considered for both branches.

The aim of the present work is a determination the exchange anisotropy role in spin nematic state formation.

1. THE GROUND STATE AND PHASE DIAGRAM

The Hamiltonian of spin-1 anisotropic crystal magnet with the nearest neighbors interaction is given by [2, 4]:

$$H = -\frac{J_1}{2}\sum_{\mathbf{n},\mathbf{m}} \mathbf{S_n S_m} - \frac{J_2}{2}\sum_{\mathbf{n},\mathbf{m}} \left(\mathbf{S_n S_m}\right)^2 + \frac{B}{2}\sum_{\mathbf{n},\mathbf{m}} \mathbf{S}_\mathbf{n}^z \mathbf{S}_\mathbf{m}^z . \quad (1)$$

Here the parameters J_1 and J_2 define the bilinear and the biquadratic exchange interaction between the nearest neighboring spins; $\hat{\boldsymbol{S}}_{\boldsymbol{n}}$ is the spin operator at site $\boldsymbol{n}$; the constant B defines the anisotropy of spins interaction (exchange anisotropy).

It is convenient to use the generalized coherent states to describe the nematic phase [5, 6]:

$$\left|\mathbf{u},\mathbf{v}\right\rangle = \sum_{j=x,y,z} \left(u_j + iv_j\right)\left|\psi_j\right\rangle, \quad (2)$$

where $\mathbf{u}$ and $\mathbf{v}$ are the real vectors; $\left|\psi_x\right\rangle = \left(\left|-1\right\rangle - \left|+1\right\rangle\right)/\sqrt{2}$, $\left|\psi_y\right\rangle = i\left(\left|-1\right\rangle + \left|+1\right\rangle\right)/\sqrt{2}$, $\left|\psi_z\right\rangle = \left|0\right\rangle$ are the usual states with spin projection $S_z = \pm 1, 0$. Taking into account the condition of normalization and the phase factor arbitrariness, these vectors satisfy two conditions:

$$\mathbf{u^2} + \mathbf{v^2} = 1, \qquad \mathbf{u}\cdot\mathbf{v} = 0. \quad (3)$$

In terms of the variable vectors $\mathbf{u}$ and $\mathbf{v}$, the vector of average spin values and the quadrupolar average ones are given by the equations [5, 6]:

$$\begin{aligned} &\left\langle \boldsymbol{S} \right\rangle = \boldsymbol{m} = 2\left[\mathbf{uv}\right], \\ &\left\langle S_i S_k + S_k S_i \right\rangle = 2\left(\delta_{ik} - u_i u_k - v_i v_k\right). \end{aligned} \quad (4)$$

The average Hamiltonian's (1) value on the states (2) defines the system energy in the mean-field approximation built with subsequent consideration of the quantum properties of the spin-1 system. In terms of the variable vectors $\mathbf{u}$ and $\mathbf{v}$, given on different lattice sites, this energy is determined by the sum on pairs of the nearest neighbors:

$$W(\mathbf{u},\mathbf{v}) = W_{is}(\mathbf{u},\mathbf{v}) + W_a(\mathbf{u},\mathbf{v}). \tag{5}$$

Here

$$W_{is} = 2\left(J_2 - J_1\right)\sum_{\mathbf{n},\mathbf{m}}\left[\left(\mathbf{u_n u_m}\right)\left(\mathbf{v_n v_m}\right) - \left(\mathbf{u_n v_m}\right)\left(\mathbf{v_n u_m}\right)\right] - \\ -\frac{J_2}{2}\sum_{\mathbf{n},\mathbf{m}}\left[\left(\left(\mathbf{u_n u_m}\right) + \left(\mathbf{v_n v_m}\right)\right)^2 + \left(\left(\mathbf{u_n v_m}\right) - \left(\mathbf{v_n u_m}\right)\right)^2\right] \tag{6}$$

is the isotropic part of the energy, which describes the isotropic spin nematic state and has been received in work [1], and

$$W_a = 2B\sum_{\mathbf{n},\mathbf{m}}\left(u_{\mathbf{n},x}v_{\mathbf{n},y} - u_{\mathbf{n},y}v_{\mathbf{n},x}\right)\left(u_{\mathbf{m},x}v_{\mathbf{m},y} - u_{\mathbf{m},y}v_{\mathbf{m},x}\right) \text{-} \tag{7}$$

is the anisotropic part of the energy.

Suppose that $\mathbf{u_n} = \mathbf{u_m} = \mathbf{u}$ and $\mathbf{v_n} = \mathbf{v_m} = \mathbf{v}$, we will get the one-site part of the energy, that defines possible homogeneous system states.

$$W = -\frac{J_2}{2} + \frac{1}{2}\left(J_2 - J_1\right)\left(m_x^2 + m_y^2\right) + \frac{1}{2}\left(J_2 - J_1 + B\right)m_z^2. \tag{8}$$

It is obvious that the nematic phase is stable at $J_2 - J_1 > 0$ and $J_2 - J_1 + B > 0$. If, at least one of these constants combinations (8) is negative, then the nematic phase loses the stability as to transition into ferromagnetic phase with $\mathbf{m}^2 = m_x^2 + m_y^2 + m_z^2 = 1$. It is easy to show that the ferromagnetic easy-axis phase is realized at $B < 0$, and the ferromagnetic easy-plane phase is stable at $B > 0$. In the limit case of the isotropic magnet at $B = 0$, and at $J_2 - J_1 < 0$ the isotropic ferromagnet state exists. The magnetic phase diagram for $S = 1$ and with the account of the exchange anisotropy is given in the picture below.

As the energy in the nematic phase does not depend on the exchange anisotropy constant, the direction of the vector-director remains completely degenerated as well as in the isotropic nematic. Thus, the exchange anisotropy does not remove the degeneracy of the ground state, as well as the single-ion anisotropy [4]. However, the exchange anisotropy removes the degeneracy of two branches of the spectrum which will be show below.

2. SPECTRUM OF THE SPIN NEMATIC

Within the frames of mean-field approximation the dynamics of the magnet with Hamiltonian (1) is described by the Lagrangian [5]

$$L = -2\hbar\sum_{\mathbf{n}}\mathbf{v_n}\partial\mathbf{u_n}/\partial t - W\left(\mathbf{u},\mathbf{v}\right). \tag{9}$$

It is necessary to take into account small transverse deviations of the vectors $\mathbf{u}$ and $\mathbf{v}$ from the ground state to find the magnon spectrum. Because of the exchange anisotropy the system has dedicated direction, and the spectrum of the system depends on the vector-director orientation in respect to the anisotropy axis.

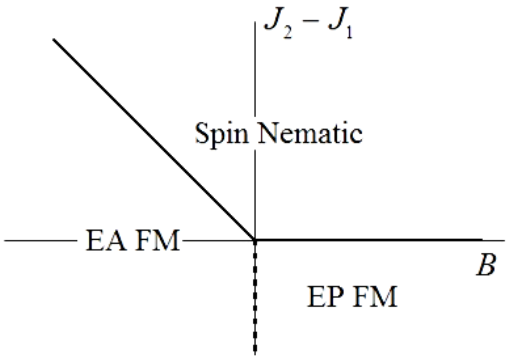

Fig. 1. Phase diagram of the anisotropic magnet in model (1). Continuous lines correspond to the loss of the stability of the spin nematic state (Spin Nematic) as to the transition into the easy-plane ferromagnetic state (EP FM, at $B > 0$), or into the easy-axis ferromagnetic state (EA FM, at $B < 0$); the dot line $B = 0$ at $J_1 > J_2$ corresponds to the degeneracy of the isotropic ferromagnetic state.

It is convenient to introduce the new coordinate system $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ in which the axis $\mathbf{e}_3$ coincides with the direction of the vector-director:

$$\begin{aligned} \mathbf{e}_1 &= \mathbf{e}_x, \\ \mathbf{e}_2 &= -\mathbf{e}_y \cos\alpha + \mathbf{e}_z \sin\alpha, \\ \mathbf{e}_3 &= \mathbf{e}_y \sin\alpha + \mathbf{e}_z \cos\alpha. \end{aligned} \tag{10}$$

Further, let us re-define the vectors $\mathbf{u}$ and $\mathbf{v}$ in terms of the Cartesian components of the small deviations from the ground state

$$\begin{aligned} \mathbf{u}_\mathbf{n} &= \left(u_{\mathbf{n},1}, u_{\mathbf{n},2}, 1 - u_{\mathbf{n},3}\right), \\ \mathbf{v}_\mathbf{n} &= \left(v_{\mathbf{n},1}, v_{\mathbf{n},2}, v_{\mathbf{n},3}\right). \end{aligned} \tag{11}$$

The Lagrangian is shown in the form of decomposition on these deviation’s degrees. For calculating the spectrum, we can restrict ourselves with the account of only the quadratic terms of small deviations $u_{\mathbf{n},1}, u_{\mathbf{n},2}$ and $v_{\mathbf{n},1}, v_{\mathbf{n},2}$. The vectors $u_{\mathbf{n},1}, u_{\mathbf{n},2}$ have sense of the generalized coordinates of the system, and $-2\hbar v_{\mathbf{n},1}, -2\hbar v_{\mathbf{n},2}$ correspond to canonical impulses [3]. So, if we have exchange anisotropy, then there are two branches the transverse oscillations of vector-director in spin nematic which are polarized in perpendicular directions.

We have to go to Hamilton formalism to describe the non-equilibrium thermodynamics.

It is convenient to realize this transition on the basis of Lagrangian, similar to work [3]. As the result the Hamiltonian is diagonal and represents the Hamiltonian of non-interacting magnons:

$$H = \sum_{\mathbf{k}} \varepsilon_1(k) a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}} + \sum_{\mathbf{k}} \varepsilon_2(k) b^{\dagger}_{\mathbf{k}} b_{\mathbf{k}}. \tag{12}$$

Here $a^{\dagger}_{\mathbf{k}}, a_{\mathbf{k}}$ and $b^{\dagger}_{\mathbf{k}}, b_{\mathbf{k}}$ are the operators of creation and annihilation of the magnons which are polarized along $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively; the $\varepsilon_1(k)$ and $\varepsilon_2(k)$ are their dispersion laws:

$$\varepsilon_1(\mathbf{k}) = z\sqrt{J_2(1-c(\mathbf{k}))}\sqrt{J_2(1-c(\mathbf{k})) + 2(J_2 - J_1 + B\sin^2\alpha)c(\mathbf{k})},$$
$$\varepsilon_2(\mathbf{k}) = z\sqrt{J_2(1-c(\mathbf{k}))}\sqrt{J_2(1-c(\mathbf{k})) + 2(J_2 - J_1)c(\mathbf{k})}.$$

(13)

Here $c(\mathbf{k}) = (1/z)\sum_{\mathbf{a}} \exp(i\mathbf{ka})$, z is the number of nearest neighbors; $\mathbf{a}$ is the radius-vector of the nearest neighbors in lattice, which is cubic.

The dispersion laws are activationless for the small wave-vectors $ak \ll 1$:

$$\begin{aligned}\varepsilon_1(\mathbf{k}) &= k\sqrt{J_2}\sqrt{2z(J_2 - J_1 + B\sin^2\alpha) + (-J_2 + 2J_1 - 2B\sin^2\alpha)a^2k^2},\\ \varepsilon_2(\mathbf{k}) &= \sqrt{J_2}\sqrt{2z(J_2 - J_1) + (-J_2 + 2J_1)a^2k^2}.\end{aligned} \tag{14}$$

The dispersion laws of both branches of spectrum are linear on the wave-vector in the whole region of stability.

$$\begin{aligned}\varepsilon_1(\mathbf{k}) &= \hbar c_1(\alpha)k, \qquad c_1(\alpha) = (a/\hbar)\sqrt{2zJ_2(J_2 - J_1 + B\sin^2\alpha)};\\ \varepsilon_2(\mathbf{k}) &= \hbar c_2 k, \qquad c_2 = (a/\hbar)\sqrt{2zJ_2(J_2 - J_1)}.\end{aligned} \tag{15}$$

Thus, the magnons, which are polarized along $\mathbf{e}_1$ and $\mathbf{e}_2$, have different velocities of propagation. The exception is the case when the vector-director is parallel to the anisotropy axis z. Anisotropy affects the velocity of only those magnons which are polarized in the direction perpendicular to the plane containing the anisotropy axis and vector-director.

4. THE ACCOUNT OF THE FLUCTUATION

Usually, within the frames of phenomenological theory the account of the anisotropy leads to a definite magnetic phase already at the $T = 0$ (for example, the easy-plane phase or the easy-axis phase). Moreover, this state exists at limit low temperatures. In other words, thermal fluctuation addition to energy is small at quite low temperatures, and it may be neglected. But in our case its account is important. In fact, at the temperature $T = 0$, the energy of the spin nematic does not depend on vector-director direction. But even elementary the addition, due thermal magnon, depends on the angle α.

The addition to energy ΔW_i, appearing from every magnons branch contribution, which is considered as ideal gas, is given by:

$$\Delta W_i = \int \varepsilon_i\left(k\right) n_{k,i} d^3 k, \qquad (16)$$

where $n_{k,i}$ is Bose's function of magnons for the given branch, $i = 1, 2$; $1/n_{k,i} = \exp[\varepsilon_i\left(\mathbf{k}\right)/T] - 1$. At low temperatures, $T << \max(J_1, J_2, B)$, the integral in (16) can be easily calculated, and its value equals $\left(\pi^4/15\right)T^4/\hbar^4 c_i^3$. Thus, the value ΔW certainly depends on α, it is less for that magnons branch for which the magnons velocity is larger. It means that the contribution in mode energy with $i = 1$ for which $c = c_1\left(\alpha\right)$ is minimal, if the angle α between the anisotropy axis and the vector-director is $\pi/2$.

CONCLUSION

Thus, while accounting the thermal additions, related to the magnons of the first branch, for the vector-director $\mathbf{u}$ the marked direction appears. If $B > 0$, then the vector-director $\mathbf{u}$ is perpendicular to the anisotropy axis due to thermal additions in the ground state, that is, the spin nematic has the easy-plane anisotropy. In this case, the spontaneous breaking of system's symmetry in respect to the vector-director $\mathbf{u}$ orientation in xy easy-plane is preserved. If $B < 0$, then the state with value $\sin^2\alpha = 0$ is preferable, and then $\mathbf{u} = \pm\mathbf{e}_z$. As the states with $\mathbf{u} = \mathbf{e}_z$ and $\mathbf{u} = -\mathbf{e}_z$ are not physically distinguished, at $B < 0$ and non-zero temperature the spontaneous breaking of system's symmetry is absent, and nematic state is not realized.

In conclusion it should be noted that one should expect another effect, then the Goldstone's behavior of magnons damping in the vicinity of softening points.

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Бутрім В. І. Вплив магнітної анізотропії на формування властивостей спінового нематика / В. І. Бутрім, Д. Д. Моісеєнко // Вчені записки Таврійського національного університету імені В. І. Вернадського. Серія : Фізико-математичні науки. – 2013 – Т. 26 (65), № 2. – С. 53-59.

В роботі досліджена модель анізотропного негейзенбергівського магнетика зі спином одиниця та обмінною анізотропією. Показано, що в системі реалізується стан анізотропного спінового нематика. Визначені умови стійкості цієї фази та кут орієнтації вектора-директора. Обчислені закони дисперсії для магнонів з різною поляризацією у фазі анізотропного спінового нематика.

Ключові слова: обмінна анізотропія, анізотропний спіновий нематик, закон дісперсії.

Бутрим В. И. Влияние магнитной анизотропии на формирование свойств спинового нематика / В. И. Бутрим, Д. Д. Моисеенко // Ученые записки Таврического национального университета имени В. И. Вернадского. Серия : Физико-математические науки. – 2013. – Т. 26 (65), № 2. – С. 53-59.
В работе исследована модель анизотропного негейзенберговского магнетика со спином единица и обменной анизотропией. Показано, что в системе реализуется состояние анизотропного спинового нематика. Определены условия устойчивости этой фазы и угол ориентации вектора-директора. Вычислены законы дисперсии для магнонов различных поляризаций в фазе анизотропного спинового нематика.
Ключевые слова: обменная анизотропия, анизотропный спиновый нематик, закон дисперсии.

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