Ученые записки Таврического национального университета имени В.И. Вернадского
Серия «Физико-математические науки». Том 25 (64). 2012 г. № 1. С. 87-94

УДК 525:641/657

GENERATION OF THE HIGH-ORDER BESSEL VORTEX-BEAM IN MONOCHROMATIC AND POLYCHROMATIC LIGHT VIA THE AXICON-UNIAXIAL CRYSTAL SYSTEM

Alexeyev A.N.

National Taurida V. Vernandsky University, Simferopol, Crimea, Ukraine
E-mail: aalexua@ukr.net

Nowhere days, a Bessel beams are of a great interest because of their high importance in practical applications in micromanipulations of micro particles and possibility to create a bottle-beams which employed in metal-cutting machines. Angular spectrum of a Bessel beam formed by a variety of a plane waves, belong to a hollow conical surface with the angle 2β at the top of the cone. Take into account this fact, one can form a Bessel beam by means of the conical lense-axicon. Generally speaking, we must note, that such a beam, formed by axicon changes it is shape along the propagation axicon and become the beam corresponding to Bessel-Gaussian solution.

***Keyword*:** Bessel beams, Bessel-Gaussian beams, axicon, monochromatic light, polychromatic light, axicon-uniaxial crystal.

INTRODUCTION

Let us consider micro particles manipulation. For that purpose we need to have the intensity minimum on the beam propagation axe. For that reason we may use the beams obtained due to the diffraction of the Gaussian beam on the computers synthesized hologram of an optical vortex. But the efficiency of this method not applicable for non-monochromatic light and low coherent light.

The other way of hollow-beams generation applicable bought to polychromatic and monochromatic light is the way to pass circularly polarized light through a uniaxe crystal along its optical axe. While this, One can obtain optical vortex in the orthogonal polarization, heaving the topological charge differs by 2 units from the charge of the initial beams. This works both to monochromatic and polychromatic beams.

The purpose of the work present is to make the future analysis of the Bessel-Gaussian beams generation by means of the axicon-uniaxial crystal both to monochromatic and polychromatic light.

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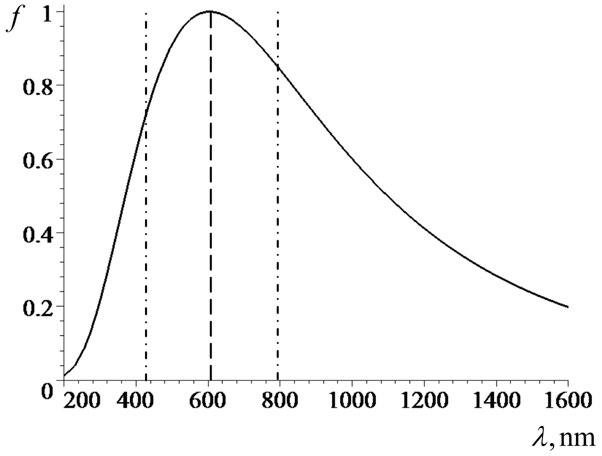

Fig. 1. Normalized spectral distribution in dependents from wave length of the light source. Visible bandwidth plotted in a dash-dote lines, intensity maximum plotted in dash line λ = 604 nm.

Such a beams can be represented by the superposition of an ordinary and extraordinary beams, with linear polarized, azimuthally and orthogonal components:

$$E_{+,BG}^{l,(o,e)} = \frac{1}{\sigma_{o,e}} J_l\left(-i\frac{K_{o,e}}{z_{o,e}\sigma_{o,e}}r\right)\exp\left\{-\frac{r^2}{w_0^2\sigma_{o,e}} - il\varphi - \frac{K_{o,e}^2}{2k_{o,e}z_{o,e}\sigma_{o,e}} + \frac{K_{o,e}^2}{2k_{o,e}z_{o,e}}\right\} \qquad (1)$$

where $K_{o,e}$ is generally a complex parameter, $\exp\left\{K_o^2/\left(2k_oz_o\right)\right\}$ the normalizing factor, $\sigma_{o,e} = 1 - iz/z_{o,e}$, $z_o = k_o w_0^2/2$, $z_e = k_e w_0^2/2$, $k_o = n_o k$, $k_e = \left(n_3^2/n_o^2\right)k_o$, k is a wave number in a free space, w_0 is a waist of beam at the plane $z = 0$. In order for the data mode beam at the entrance face of the crystal form a single polarized beam right circularly Bessel-Gauss, it is necessary that the field mode beam at the boundary agreed. This condition implies that $K_e = \frac{n_e}{n_o}K_o$.

Lets consider that initial beam has CW polarization, that it's circularly polarized components will be written as:

$$
\begin{aligned}
E^{l,+}_{+,BG} &= \exp\left(-il\varphi\right)\left\{J_l\left(-i\frac{K_o}{z_o\sigma_o}r\right)\exp\left(-\frac{K_o^2}{2k_oz_o\sigma_o}\right)\Psi_o + \right. \\
&\left. +J_l\left(-i\frac{K_o}{z_o\sigma_e}r\right)\exp\left(-\frac{K_o^2}{2k_oz_o\sigma_e}\right)\Psi_e\right\}\exp\left(\frac{K_o^2}{2k_oz_o}\right),
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
E^{l,+}_{-,BG} &= \left(\frac{-iK_o}{k_ow_0}\right)^l\left(\frac{re^{-i\varphi}}{w_0}\right)^l\frac{w_0^2e^{i2\varphi}}{r^2}\sum_{j=0}^{\infty}\frac{\left(j+1\right)}{\left(j+l\right)!}\left(\frac{-K_o^2}{2k_oz_o}\right)^j \times \\
&\times\left\{\frac{\Psi_o}{\sigma_o^{j+l-1}}L_{j+1}^{(l-2)}\left(\frac{r^2}{w_0^2\sigma_o}\right)-\frac{\Psi_e}{\sigma_e^{j+l-1}}L_{j+1}^{(l-2)}\left(\frac{r^2}{w_0^2\sigma_e}\right)\right\}\exp\left(\frac{K_o^2}{2k_oz_o}\right).
\end{aligned}
$$

It has to be notified, that CCW polarized field component can be represented as a super position of an infinite series of the Lager-Gaussian beams of a complex argument function. It seems to be, that this super position might not have the field envelope in the form of Bessel function, never the less comparison of such the field and the Bessel field shows good match, accurate to the amplitude coefficient (Fig. 2).

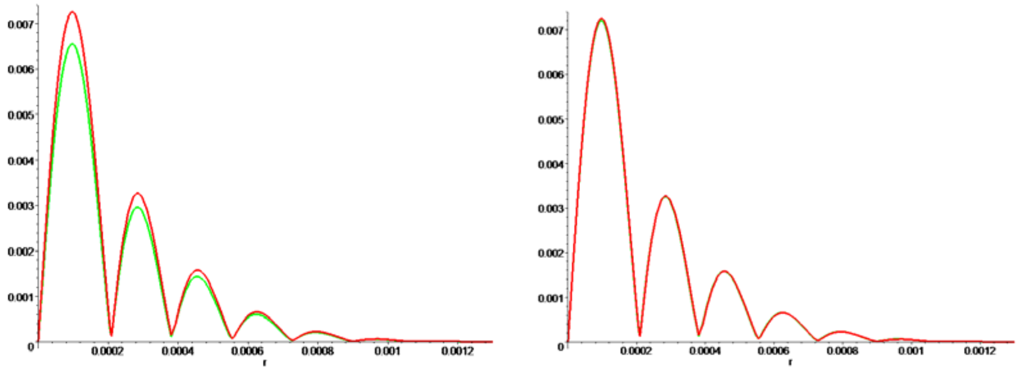

Fig. 2. Comparison between CCW Bessel-Gauss and second order Bessel beam polarization component in the crystal.

Let us consider the Bessel-Gaussian beam evolution along the crystal. As it might seen at the picture, at the relatively small distances Gaussian field envelope has an oscillations specified by the Bessel function modulation. On increasing the light propagation length in crystal, this oscillation decreases and one can observe dipper minima on the axis. Topological charge of the CCW beam component modulo 2 less than CW beam component charge, because we choose negative topological charge of the initial beam. The evolution of the Bessel-Gaussian beam formed by axicone shown on (Fig. 2.) As one can see, the beam with topological charge to generates in CCW beam component.

To calculate the propagation of the Bessel-beam in crystal we used following method.

We use the black body with color temperature 4800 K as a light source.

RGB components of the monitor brightness were calculated according to the following relations:

$$R=\left[\int f(\lambda)I(x,y,z,\lambda)\overline{r}(\lambda)d\lambda\right]^{1/\chi},$$

$$G=\left[\int f(\lambda)I(x,y,z,\lambda)\overline{g}(\lambda)d\lambda\right]^{1/\chi}, \qquad (3)$$

$$B=\left[\int f(\lambda)I(x,y,z,\lambda)\overline{b}(\lambda)d\lambda\right]^{1/\chi},$$

where $I(x,y,z,\lambda)=\left|E(x,y,z,\lambda)\right|^2$ intensity distribution, $\overline{r},\overline{g},\overline{b}$ are coordinates of color in the spectrum the integration is performed in the same range of wavelengths. χ - the nonlinearity of the monitor. As the table is set, then the integration in (3) is replaced by a summation over 81 point in increments. If some of the integrals (3) are negative, which means that the corresponding colors are outside the gamut of the monitor, and data components of the color (negative), we assumed to be zero. In the calculations it must also be taken into account the dispersion of the refractive indices of a uniaxial crystal. Fig. 3 shows this dependence in the visible range for the lithium niobate crystal, and the effect of photo refraction we have not taken into account.

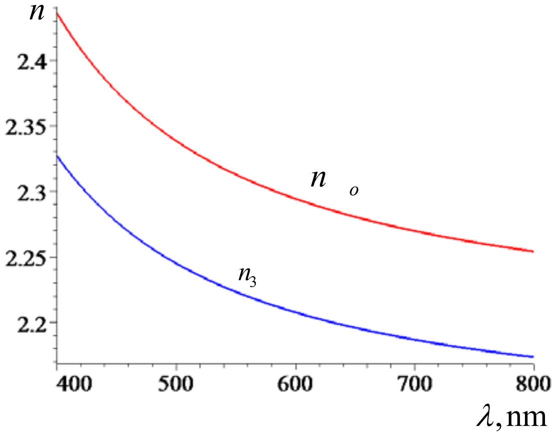

Fig. 3. The spectral dependence of the refractive index of the wavelength of visible range in lithium niobate.

We change integration to the summing by 81 points in optical bandwidth, having RGB spectrum coordinates, set by specialized colorimetric table.

The color distributions in the circularly polarized beam components shown on (Fig. 4).

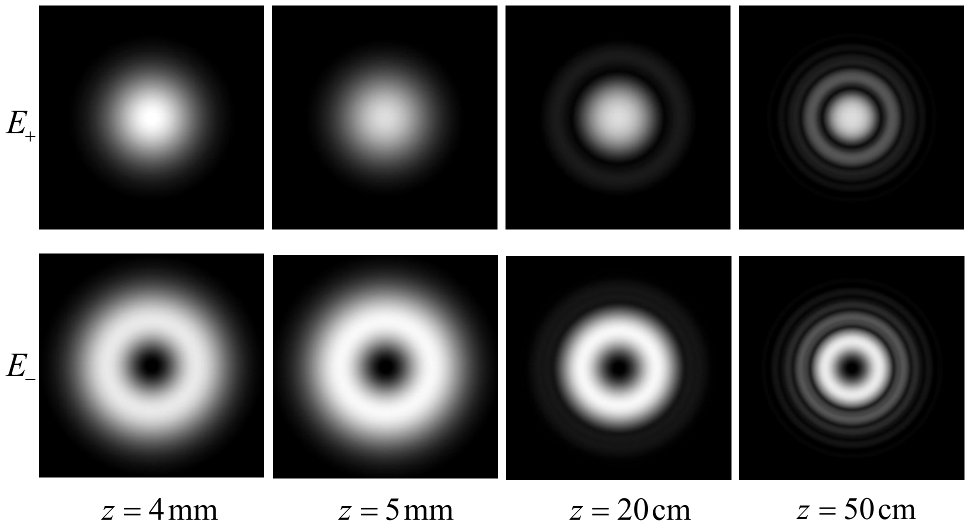

Fig. 4. Generating optical vortex with a double topological charge in the white light $(l = 0, n = 0)$ with $w_0 = 20\mu$m in a lithium niobate crystal, the image size on the axes x, y: $5w$, $w = w_0\sqrt{1+(z/z_0)^2}$.

As it seen, there is an optical vortex of charge to in the middle of the CCW beam component.

We made the following experimental set up to generate the Bessel-Gaussian beams.

The light from He-Ne laser passes through axicon, than it collimated by lens and becomes CCW polarized after polarizer and λ/4 plate. After this, this beam focuses on the incoming fringe of the $LiNiTiO_3$ crystal, having its axis along the beam. The beam passed through the crystal collimates by next lens and with the aide of polarizer and λ/4 plate we separates CCW Bessel-Gauss beam component with topological charge two. For the generation of the higher order Bessel-Gaussian beams we only need to duplicate the elements put in the dashed block (Fig. 5).

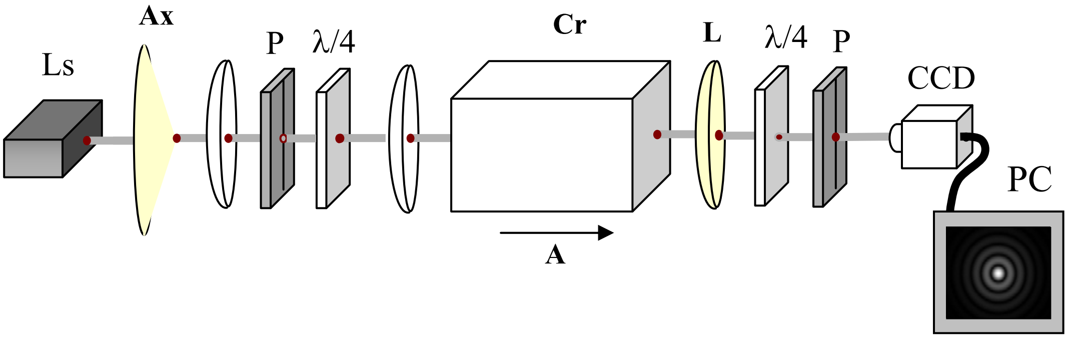

Fig. 5. Experimental setup for obtaining a monochromatic beam of Bessel-Gauss: Ls – laser, P – polarizer, λ/4 – quarter-wave plate, Ax – axicon, L – lens, Cr – $LiNbO_3$ crystal A – symmetry axis, CCD – CCD camera, PC – computer.

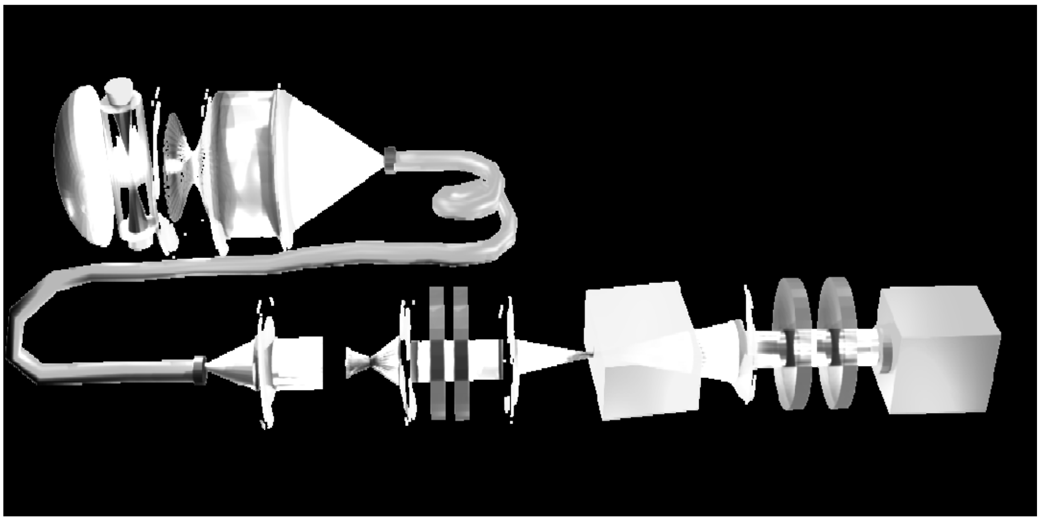

Fig. 6. The experimental setup for generation of polychromatic Bessel-Gauss beam.

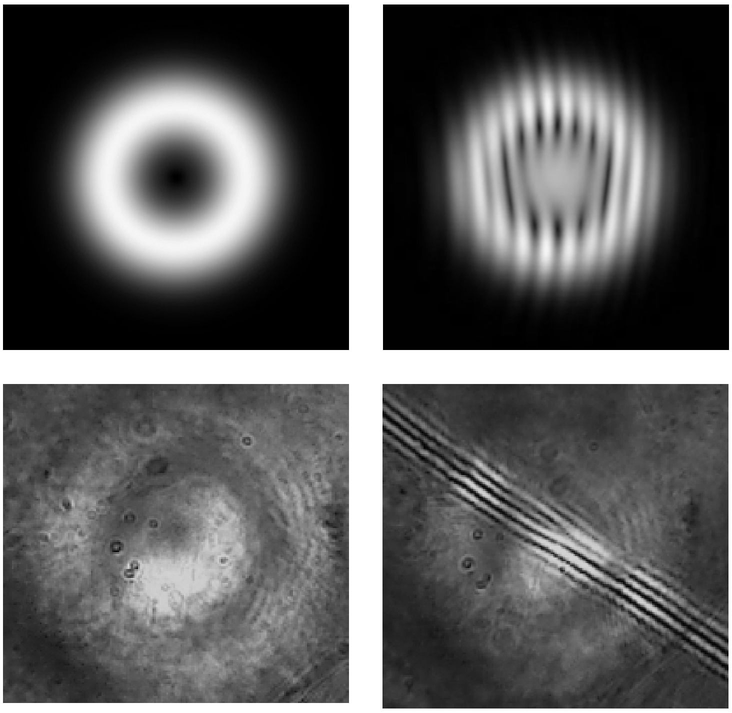

Fig. 7. Generation of "white" Bessel-Gauss beam with *l*=1.

We have employed the property of the axicon- uniaxial crystal system to generate the second-order vortex-beam both in monochromatic and polychromatic light. We have performed two groups of experiments when the initial Gaussian beam passes at first through the uniaxial crystal and then the axicon and vice-versa, each experiment being accompanied by testing the obtained beam structure to slightest perturbations of the elements of the experimental set-up. We have revealed that the most reliable optical scheme both for polychromatic and monochromatic light is that consisting of the sequence: axicon-crystal rather than the crystal-axicon. The experiments were complemented by the interference patterns of the white-vortex- Bessel beams of the high-orders (Fig. 6).

At the picture below (Fig. 7) we represents the experimental results of the first order Bessel-Gaussian beam generation and there comparison with the computer simulation.

CONCLUSIONS

At the paper present it was theoretically reveled and experimentally proved that color zero order Bessel-Gaussian beam could be generated by means of thermal light source having spatial coherence rather then temporal by means of conical refractive surface-axicone. It was theoretically predicted and experimentally proved that one can obtain color second order Bessel-Gaussian beam generation in orthogonal circular polarized component of initial beam passed through uniaxial crystal along it's optical axis. The system of uniaxial crystal-circular polarizer can be doubled thus charge also will be doubled.

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Алексєєв О. М. Генерація вихрових пучків Бесселя-Гаусса вищих порядків в поліхроматичному світлі за допомогою системи аксікон-одноосьовий кристал / Алексєєв О. М. // Вчені записки Таврійського національного університету імені В.І. Вернадського. Серія: Фізико-математичні науки. – 2012. – Т. 25(64), № 1. – С. 87-94.

У роботі показано, що при поліхроматичного пучок Бесселя-Гаусса нульового порядку може бути отриманий при пропусканні випромінювання HID лампи через одномодове волокно, на якому генерується гаусів пучок, і конічну лінзу (аксікон). Теоретично обгрунтовано та експериментально показано, що при проходженні циркулярно поляризованого пучка Бесселя-Гаусса через одноосьовий кристал вздовж його оптичної осі, в ортогонально циркулярно поляризованої компоненті генерується пучок Бесселя-Гаусса другого порядку в білому світі. При пропущенні такого пучка через каскад одноосьових кристалів з розташованими між ними циркулярно поляризованими фільтрами (складається з ахроматичний пластини і лінійного поляризатора) генерується пучок Бесселя-Гаусса вищого порядку з парним індексом. Такий пучок можна використовувати в високопотужних оптичних спанерах для захоплення і транспортування мікрочастинок.

Ключові слова: пучок Бесселя, пучок Бесселя-Гаусса, аксікон, монохроматичне світло, поліхроматичне світло, аксікон-одноосьовий кристал.

Алексеев А. Н. Генерация вихревых пучков Бесселя-Гаусса высших порядков в полихроматическом свете по средством системы аксикон-одноосный кристалл / Алексеев А. Н. // Ученые записки Таврического национального университета имени В.И. Вернадского. Серия: Физико-математические науки. – 2012. – Т. 25(64), № 1. – С. 87-94.

В работе показано, что при полихроматический пучок Бесселя Гаусса нулевого порядка может быть получен при пропускании излучения HID лампы через одномодовое волокно, на котором генерируется гауссов пучок и коническую линзу (аксикон). Теоретически обосновано и экспериментально показано, что при прохождении циркулярно поляризованного пучка Бесселя-Гаусса через одноосный кристалл вдоль его оптической оси, в ортогонально циркулярно поляризованной компоненте генерируется пучок Бесселя-Гаусса второго порядка в белом свете. При пропускании такого пучка через каскад одноосных кристаллов с расположенными между ними циркулярно поляризованными фильтрами (состоит из ахроматической пластины $\lambda/4$ и линейного поляризатора) генерируется пучок Бесселя-Гаусса высшего порядка с четным индексом. Такой пучок можно использовать в высокомощных оптических спанерах для захвата и транспортировки микрочастиц.

Ключевые слова: пучок Бесселя, пучок Бесселя-Гаусса, аксикон, монохроматический свет, полихроматический свет, аксикон-одноосный кристалл.

Поступила в редакцию 11.04.2012 г.