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COUPLED MODES THEORY FOR PERTURBED SPUN OPTICAL FIBRES

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We developed a modification of the coupled mode theory for Schrödinger-type equations with periodic potentials in the presence of an invariant perturbation. The scheme is applied to obtaining coupled mode equations for perturbed spun optical fibres.
Key words: coupled mode theory, spun optical fibres.

INTRODUCTION

The development of singular optics has unveiled an extreme relevance of optical vortices (OV) to the problem of increasing the "information capacity" of a signal [1-7]. However, free-space OV-based communication, along with other factors, suffers from aberrations due to atmosphere turbulence [8], which results in scattering the initial OAM state into the whole set of OAM states. In this regard communication via optical waveguides seems to be more protected from perturbations. It has been suggested to use for OV-based fibre communications the spun optical fibres, in which either the director of induced transverse anisotropy or the major axis of the deformation ellipse regularly rotate with z increasing [9, 10]. As has been shown, the modes of such may be circularly (CP) or linearly (LP) polarized OVs.

The question of stability of OVs with respect to external perturbation in such fibres, however, has not been solved. It turned out that the existing methods of treating simultaneous action of z-variant and z-invariant perturbations are inapplicable, at least without essential modifications, to this problem. In this connection the aim of the present paper is to develop a modification of the coupled mode theory, which may enable one to study the influence of z-invariant perturbation on the solutions of Schrödinger-type equations with z-periodic potentials. The developed method can be applied, in particular, to the study of robustness of OVs in spun fibres to external z-independent perturbations.

1. MODIFIED COUPLED MODES THEORY

Though application of perturbation theory method proved to be convenient and reliable for the solution of the problem of regularly spun fibres, in the case where the refractive index is given by the sum of a periodic and a z-independent function, it seems impossible to obtain any eigenvalue equation of the type suggested in [9, 10]. In such a situation it is natural to try to make use of another powerful (and more conventional) method – the coupled mode theory (CMT). However, its classical variant where the zero-order Hamiltonian is assumed to be z-independent and the perturbation is given by some periodic in *z* function [11], should be modified to meet the needs of our particular problem. Indeed, in our case the modes of twisted fibres are formed by the periodic refractive index

$n(x,y,z)$, whereas the perturbation term $\Delta n(x,y)$ is translational invariant. As is known, in the scalar approximation, which seems to be sufficient for the description of vortex mode regime, where the spin-orbit interaction (SOI) is suppressed by the twisting, the transverse electric field $\mathbf{E}$ (for simplicity we omit the subscript "t") satisfies the equation [11, 12]:

$$\Delta + k^2[n^2(x,y,z) + \Delta n^2(x,y)]\mathbf{E}(x,y,z) = 0 \ . \qquad (1)$$

Here Δ is the Laplace operator, $k = 2\pi/\lambda$ and λ is the wavelength in vacuum. The regular refractive index n^2 describes the effect of twisting combined with the refractive index distribution $\tilde{n}^2(x,y)$ of an ideal fibre. For anisotropic twisted fibres one has [9]:

$$n^2(x,y,z) = \tilde{n}^2(x,y) + \Delta n^2 \begin{pmatrix} 0 & \exp(-2iqz) \\ \exp(2iqz) & 0 \end{pmatrix}, \qquad (2)$$

where $\Delta n^2 = (n_e^2 - n_o^2)/2$ and n_e^2, n_o^2 are principal values of transverse refractive index tensor, $q = 2\pi/H$, H is the twist pitch. As usual, $\tilde{n}^2(x,y) = n_{co}^2(1 - 2\Delta f(x,y))$ where n_{co} is the refractive index in the core, Δ is refractive index contrast, f is the profile function [12]. Note that in (2) we use representation in the basis of circular polarizations, where $E_{\pm} = \left(E_x \mp iE_y\right)/\sqrt{2}$. Refractive index in elliptical twisted fibres is given by [10]:

$$n^2(x,y,z) = \tilde{n}^2(x,y) - 2n_{co}^2 \Delta\delta f_r^{/} \cos 2(\varphi - qz), \qquad (3)$$

where cylindrical polar coordinates (r,φ,z) are implied, $\delta << 1$ is the ellipticity parameter.

Analogously to the standard variant of the CMT we start from the notion that the solutions $\mathbf{E}_m$ of zero-order equation are known:

$$\Delta + k^2 n^2(x,y,z)\mathbf{E}_m(x,y,z) = 0 \ . \qquad (4)$$

In contrast to the standard CMT scheme, here the dependence of $\mathbf{E}_m$ on z does not reduce to a simple multiplication by a factor $\exp(i\tilde{\beta}z)$. We search for the solutions of the equation for a perturbed fibre:

$$\left\{\Delta + k^2[n^2(x,y,z) + \Delta n^2(x,y)]\right\}\mathbf{E}_m(x,y,z) = 0 \qquad (5)$$

in the form

$$\mathbf{E}(x,y,z) = \sum_m A_m(z)\mathbf{E}_m(x,y,z), \qquad (6)$$

where $A_m(z)$ are the slowly varying amplitudes. Substituting (5) into (6) and allowing for (4) one can get:

$$\Delta\mathbf{E} \approx \sum_m \left\{ A_m(z)\Delta\mathbf{E}_m(x,y,z) + 2\frac{\partial A_m(z)}{\partial z}\frac{\partial \mathbf{E}_m(x,y,z)}{\partial z} \right\}. \qquad (7)$$

Here, as usual, we neglected the second derivative $A_m''(z)$. In the standard variant of CMT the derivative $\mathbf{E}'_m$ is replaced by $i\beta\mathbf{E}_m$ term. In our case, however, the situation is more complicated. As follows from the results of [9], the dependence of the modes $\mathbf{E}_m$ on z is more intricate.

Indeed, as follows from the results of [9], modes of spun anisotropic fibres in case vortex-mode regime is implemented are given (for the set with orbital number $l = 1$) by LP OVs, whose polarization adiabatically traces the direction of local anisotropy axes:

$$\mathbf{\Psi}_1 = e^{i\varphi}\begin{pmatrix}\cos qz\\ \sin qz\end{pmatrix}_L \exp(i\beta_+ z),\ \mathbf{\Psi}_2 = e^{-i\varphi}\begin{pmatrix}\cos qz\\ \sin qz\end{pmatrix}_L \exp(i\beta_+ z),$$
$$\mathbf{\Psi}_3 = e^{i\varphi}\begin{pmatrix}-\sin qz\\ \cos qz\end{pmatrix}_L \exp(i\beta_- z),\ \mathbf{\Psi}_4 = e^{-i\varphi}\begin{pmatrix}-\sin qz\\ \cos qz\end{pmatrix}_L \exp(i\beta_- z), \tag{8}$$

where $E = \Delta n^2 k^2$ describes initial anisotropy of the fibre, $\widetilde{\beta}$ is the scalar propagation constant and the subscript L denotes representation in the basis of linear polarizations and $\beta_{\pm} = \widetilde{\beta} \pm \Delta\beta / 2$, $\Delta\beta = E / \widetilde{\beta}$. From (8) it follows:

$$\frac{\partial \mathbf{\Psi}_1}{\partial z} = i\beta_+\mathbf{\Psi}_1 + qe^{i\Delta\beta z}\mathbf{\Psi}_3,\ \frac{\partial \mathbf{\Psi}_2}{\partial z} = i\beta_+\mathbf{\Psi}_2 + qe^{i\Delta\beta z}\mathbf{\Psi}_4,$$
$$\frac{\partial \mathbf{\Psi}_3}{\partial z} = i\beta_-\mathbf{\Psi}_3 - qe^{-i\Delta\beta z}\mathbf{\Psi}_1,\ \frac{\partial \mathbf{\Psi}_4}{\partial z} = i\beta_-\mathbf{\Psi}_4 - qe^{-i\Delta\beta z}\mathbf{\Psi}_2. \tag{9}$$

Then the first term on the right of (7) being combined with the $A_m k^2 n^2 \mathbf{E}_m$ term vanishes due to (4) and one arrives at the standard equation:

$$\sum_m \left\{ 2\frac{\partial A_m(z)}{\partial z}\frac{\partial \mathbf{E}_m(x,y,z)}{\partial z} + k^2\Delta n^2(x,y)A_m(z)\mathbf{E}_m \right\} = 0, \tag{10}$$

where for the derivatives $\mathbf{E}'_m$ one should use (9) (identifying $\mathbf{\Psi}_m$ with $\mathbf{E}_m$). Using connection (9) in the form:

$$\frac{\partial \mathbf{E}_m}{\partial z} = \sum_k Q_{mk}\mathbf{E}_k, \tag{11}$$

one can bring (10) to the form:

$$\sum_{m,k}\left\{ 2\frac{\partial A_m(z)}{\partial z}Q_{mk} + k^2\Delta n^2(x,y)A_m(z)\delta_{mk}\right\}\mathbf{E}_k = 0, \tag{12}$$

from whence it follows the desired equation in slow amplitudes $A_m(z)$:

$$2i\langle l|l\rangle \sum_m \frac{dA_m}{dz}Q_{ml} = -k^2\sum_m \langle l|\Delta n^2|m\rangle A_m. \tag{13}$$

In this equation we used standard Dirac's notations, where the scalar product implies integration over the total cross-section of the fibre. This equation is the main result of the present paper and is a generalization of its more conventional form widely spread in the literature. Note that here we do not specify normalization and the phase exponentials should be included into the structure of the modes $|m\rangle$. The developed formalism can be applied to study the effect of z-invariant perturbations on the mode structure in spun fibres.

For anisotropic spun fibres the perturbation operator Δn^2 should be taken in the form:

$$\Delta n^2(x,y) = \delta n^2 \hat{\sigma}_z , \tag{14}$$

where $\hat{\sigma}_z$ is the Pauli matrix and δn^2 characterizes induced birefringence. For elliptic spun fibres one has

$$\Delta n^2 = -2n_{co}^2 \delta \Delta r f_r' \cos 2\varphi \ . \tag{15}$$

Finally, let us make a remark on application of CMT to the problems concerned with degenerate states. The classical variant of CMT implies using a periodic perturbation $\Delta n(x,y,z)$ on the background of aperiodic refractive index $n(x,y)$. Suppose that the perturbation term is also aperiodic and z-independent. In addition, let us assume that there is degeneracy in the system and the fields $\mathbf{E}_m$ propagate with the same propagation constant β. Then (13) is reduced to

$$2i\langle l|l\rangle \beta \frac{dA_l}{dz} = -k^2 \sum_m \langle l|\Delta n^2|m\rangle A_m \ . \tag{16}$$

where the vectors $|m\rangle$ have the same exponential factors so that there is no z-dependence on the right of (16). Searching for the solution in the form $A_l = C_l \exp(i\beta z)$ one readily arrives at the standard eigenvalue equation for the perturbation matrix $V_{lm} = \langle l|\Delta n^2|m\rangle$:

$$k^2 \hat{V}\mathbf{A} = 2\langle l|l\rangle \beta^2 \mathbf{A} \ . \tag{17}$$

Here the factor $\langle l|l\rangle$ reflects arbitrariness of normalization. In this way CMT can be used for standard quantum-mechanical problems in the presence of degeneracy.

CONCLUSION

In this paper we have developed a modification of the coupled mode theory for Schrödinger-type equations with periodic potentials in the presence of an invariant perturbation. The scheme is applied to obtaining coupled mode equations for perturbed spun optical fibres. The method developed may be useful while studying the question of robustness of optical vortices in spun anisotropic and elliptical fibres with respect to external perturbations, which do not depend on the longitudinal coordinate. This scheme may also present an alternative to the standard perturbation theory with degeneracy.

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Алексеєв К. М. Теорія зв'язаних мод для збурених скручених оптичних волокон / Алексеєв К. М. // Вчені записки Таврійського національного університету імені В.І. Вернадського. Серія: Фізико-математичні науки. – 2012. – Т. 25(64), № 1. – С. 75-79.
Розвинена модифікація теорії зв'язаних мод для рівнянь типу Шредингера із періодичним потенціалом у присутності інваріантного збурення. Схема прикладена до отримання рівнянь зв'язаних мод для збурених скручених оптичних волокон.
Ключові слова теорія зв'язаних мод, скручені оптичні волокна.

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Развита модификация теории связанных мод для уравнений типа шредингеровского с периодичным потенциалом в присутствии инвариантного возмущения. Схема приложена к получению уравнений связанных мод для возмущенных скрученных оптических волокон.
***Ключевые слова**:* теория связанных мод, скрученные оптические волокна.

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