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EXISTENCE OF GUARANTEED SOLUTION FOR MULTICRITERIA PROBLEM

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Conditions for guaranteed solution existence in a multicriteria dynamical problem under uncertainty are obtained.

1. STATEMENT OF THE PROBLEM

Let us consider N-criteria dynamical problem under uncertainty

$$< \Sigma, \mathcal{U}, \mathcal{V}, \{J^{(i)}(u,v)\}_{i\in \mathbf{N}} > . \tag{1}$$

The controlled system Σ changing in time t is described by the vector differential equation

$$\dot{x} = \phi(t,x,u,v), \quad x(t_*) = x_*. \tag{2}$$

The phase vector $x \in \mathbf{R}^n$, the starting moment $t_* \geq 0$ and the time of process finishing $\vartheta > t_*$ are fixed; the time $t \in [t_*, \vartheta]$. The PMD's (person making decisions) control action is $u \in \mathbf{R}^r$, an uncertainty effect is $v \in \mathbf{R}^q$. Assume that the set $\mathcal{U}$ of controls $u(t), t \in [t_*, \vartheta]$, and the set $\mathcal{V}$ of uncertainties $v(t), t \in [t_*, \vartheta]$, are convex, closed and bounded subsets of $L_2^r[t_*, \vartheta]$ and $L_2^q[t_*, \vartheta]$ respectively with nonempty interior.

Moreover let the right part of (2) be such that for all $u(\cdot) = \{u(t), t_* \leq t \leq \vartheta\} \in \mathcal{U}$ and $v(\cdot) \in \mathcal{V}$ the system (2) (where $u = u(t)$ and $v = v(t)$) has a unique solution $x(t)$ continuous and extendable on the interval $[t_*, \vartheta]$ and satisfying (2) for almost all $t \in [t_*, \vartheta]$ (you can find such restrictions for $\mathcal{U}$, $\mathcal{V}$ and $\phi(t,x,u,v)$ in [1,2]). The quality of system Σ functioning is valued by the vector criterion $J(u,v) = (J^{(1)}(u,v), ..., J^{(N)}(u,v))$, where the functionals

$$J^{(i)}(u,v) = \Phi^{(i)}(x(\vartheta)) + \int\limits_{t_*}^{\vartheta} F^{(i)}(t, x(t), u(t), v(t))dt, \quad i \in \mathbf{N} = \{1, .., N\} \tag{3}$$

are determined on the triples $(x(t), u(t), v(t))$, $t \in [t_*, \vartheta]$. Assume, that the scalar functions $\Phi^{(i)}(x), F^{(i)}(t,x,u,v)$ $(i \in \mathbf{N})$ are continuous.

The process of decision making in (1) happens as follows. PMD chooses and uses a control $u(\cdot) \in \mathcal{U}$. Irrespectively of this choice some uncertainty $v(\cdot) \in \mathcal{V}$ effects the system Σ. Then the solution $x(t)$, $t \in [t_*, \vartheta]$, is constructed for system (2) for $u = u(t)$, $v = v(t)$. On the sets $(x(t), u(t), v(t) |\ t \in [t_*, \vartheta])$ N criteria (3) are determined. In terms of "meaning"the PMD's objective point in the problem (1) is to choose his control $u(\cdot) \in \mathcal{U}$ so that all N criteria (3) take the largest possible values simultaneously. In addition when choosing his control the PMD should allow for emerging of any uncertainty $v(\cdot) \in \mathcal{V}$.

For formalization of the solution to the problem (1) we shall use the notion of optimum by Geoffrion (see [3]) from the multicriteria problem theory [4].

Definition. The couple $(u^G(\cdot), J^G) \in \mathcal{U} \times \mathbf{R}^N$ is called the *Geoffrion guaranteed solution of multicriteria problem under uncertainty* (1) if there exists an uncertainty $v^G(\cdot) \in \mathcal{V}$ such that $J^G = J(u^G, v^G)$ and

1^0 the control $u^G(\cdot) \in \mathcal{U}$ is maximal by Geoffrion in multicriteria problem

$$< \mathbf{\Sigma}(v = v^G), \mathcal{U}, J(u, v^G) >, \tag{4}$$

obtained from (1) by fixing $v(\cdot) = v^G(\cdot) \in \mathcal{V}$.

2^0 the uncertainty $v^G(\cdot) \in \mathcal{V}$ is minimal by Geoffrion in a multicriteria problem

$$< \mathbf{\Sigma}(u = u^G), \mathcal{V}, J(u^G, v) > \tag{5}$$

obtained from (1) by fixing $u(\cdot) = u^G(\cdot) \in \mathcal{U}$. The couple $(u^G(\cdot), v^G(\cdot))$ is said to be a *saddle point by Geoffrion* in the problem (1).

Remark 1. PMD using the control $u^G(\cdot) \in \mathcal{U}$ provides himself a vector guarantee J^G. Namely, whatever uncertainty $v(\cdot) \in \mathcal{V}$ is realized in the problem (1), the components of the vector criterion $J(u^G, v)$ obtained so far can not become smaller simultaneously than the corresponding components of the vector guarantee J^G. That is why the solution of the problem (1) is determined in the form of the couple $(u^G(\cdot), J^G)$.

Remark 2. According to the given definition to construct the Geoffrion guaranteed solution $(u^G(\cdot), J^G)$ for the problem (1) one must find a saddle point by Geoffrion $(u^G(\cdot), v^G(\cdot)) \in \mathcal{U} \times \mathcal{V}$ for this problem.

Remark 3. In problem (4) the system $\mathbf{\Sigma}(v = v^G)$ is of the form

$$\dot{x} = \phi(t, x, u, v^G(t)), \quad x(t_*) = x_*,$$

and criteria (3) are converted into

$$J^{(i)}(u, v^G) = \Phi^{(i)}(x(\vartheta)) + \int_{t_*}^{\vartheta} F^{(i)}(t, x(t), u(t), v^G(t))dt, \quad i \in \mathbf{N}.$$

In the problem (4) a control $u^G(\cdot) \in \mathcal{U}$ is maximal by Geoffrion if

1) $u^G(\cdot)$ is maximal by Pareto in this problem, i.e. for all $u(\cdot) \in \mathcal{U})$ the system of inequalities

$$J_i(u^G, v^G) \leq J_i(u, v^G), \quad i \in \mathbf{N}$$

is incompatible, besides at least one inequality is strict;

2) there exists a constant $\gamma_1 > 0$ such that if for some $j \in \mathbf{N}$ and $u(\cdot) \in \mathcal{U}$ we have

$$J_j(u, v^G) > J_j(u^G, v^G),$$

then the number $k \in \mathbf{N}$ is found such that

$$J_k(u, v^G) < J_k(u^G, v^G),$$

and

$$J_j(u, v^G) - J_j(u^G, v^G) \leq \gamma_1[J_k(u^G, v^G) - J_k(u, v^G)].$$

Similarly, the uncertainty $v^G(\cdot) \in \mathcal{V}$ is minimal by Geoffrion in the problem (5) if

1) it is minimal by Pareto in this problem, i.e. for all $v(\cdot) \in \mathcal{V}$ the system of inequalities

$$J_i(u^G, v) \leq J_i(u^G, v^G), \quad i \in \mathbf{N},$$

(at least one of which is strict) is incompatible;

2) there exists a constant $\gamma_2 > 0$ such that if for some $j \in \mathbf{N}$ and $v(\cdot) \in \mathcal{V}$ we get

$$J_j(u^G, v) < J_j(u^G, v^G),$$

Then one can find the number $k \in \mathbf{N}$ such that

$$J_j(u^G, v^G) - J_j(u^G, v) \leq \gamma_2[J_k(u^G, v) - J_k(u^G, v^G)].$$

2. Auxiliary Assertions

Assertion1. If there exists $\alpha_i = const > 0$ and $\beta_i = const > 0$, $i \in \mathbf{N}$, such that

$$\max_{u(\cdot)\in\mathcal{U}} \sum_{i\in\mathbf{N}} \alpha_i J_i(u, v^G) = \sum_{i\in\mathbf{N}} \alpha_i J_i(u^G, v^G),$$

$$\max_{v(\cdot)\in\mathcal{V}} [-\sum_{i\in\mathbf{N}} \beta_i J_i(u^G, v)] = -\sum_{i\in\mathbf{N}} \beta_i J_i(u^G, v^G). \quad (6)$$

Then the couple $(u^G(\cdot), v^G(\cdot)) \in \mathcal{U} \times \mathcal{V}$ is a saddle point by Geoffrion in the problem (1).

This assertion is implied by the Geoffrion Theorem [4, p.80].

Now let us consider an auxiliary non-cooperative two-person game

$$< \{1, 2\}, \mathcal{U}, \mathcal{V}, \{I_i^*(u, v)\}_{i=1,2} > . \quad (7)$$

In this game the player 1 using his strategy $u(\cdot) \in \mathcal{U}$ strives to get the largest possible value of his payoff function

$$I_1^*(u, v) = I_1^*(u, v, \alpha) = \sum_{i\in\mathbf{N}} \alpha_i J_i(u, v). \quad (8)$$

And the player 2, choosing $v(\cdot) \in \mathcal{V}$, strives to get the largest possible value of his payoff function

$$I_2^*(u, v) = I_2^*(u, v, \beta) = -\sum_{i\in\mathbf{N}} \beta_i J_i(u, v). \quad (9)$$

The Nash equilibrium situation $(u^G(\cdot), v^G(\cdot)) \in \mathcal{U} \times \mathcal{V}$ in the game (7) is determined (see [5]) by the equalities (6) (of course, if the constants $\alpha_i > 0$ and $\beta_i > 0$ $(i \in \mathbf{N})$ are given in advance).

Remark 4. Thus to construct a saddle point by Geoffrion $(u^G(\cdot), v^G(\cdot))$ one should find a Nash equilibrium situation in the game (7) for at least one set of constants $\alpha_i > 0$, $\beta_i > 0$ $(i \in \mathbf{N})$. In view of Remark 2 the constructing of Geoffrion guaranteed solution for the problem (1) is reduced to the mentioned situation constructing.

Let in the game (7) the constants $\alpha_i = \beta_i = 1$ $(i \in \mathbf{N})$. In this case we reduce the constructing of Nash equilibrium situation in (7) to the finding of saddle point in a two-person zero-sum game

$$< \{1, 2\}, \mathcal{M}, \mathcal{M}, I(U, V) > \quad (10)$$

obtained from (7). Namely, in the game (10) the player 1 choosing his strategy $U(\cdot) = (u_1(\cdot), u_2(\cdot)) \in \mathcal{M}$ strives to increase the value of a functional

$$I(U, V) = \sum_{i\in\mathbf{N}} J_i(u_1, v_2) - \sum_{i\in\mathbf{N}} J_i(v_1, u_2). \quad (11)$$

And the player 2 choosing $V(\cdot) = (v_1(\cdot), v_2(\cdot)) \in \mathcal{U} \times \mathcal{V} = \mathcal{M}$ strives to decrease mentioned functional.

The solution of the game (10) is a saddle point $(U^0(\cdot), V^0(\cdot)) \in \mathcal{M} \times \mathcal{M}$ determined by the inequalities

$$I(U, V^0) \leq I(U^0, V^0) \leq I(U^0, V), \quad \forall U(\cdot) \in \mathcal{M}, \forall V(\cdot) \in \mathcal{M}. \quad (12)$$

The functional $I(U, V)$, determined on $\mathcal{M} \times \mathcal{M}$, is called [2, p. 176] a strongly concave in $U(\cdot)$ over $\mathcal{M}$ if there exists $\tau_1 = const > 0$ such that for every $V(\cdot) \in \mathcal{M}$

$$I(\lambda U^{(1)} + (1-\lambda)U^{(2)}, V) \geq \lambda I(U^{(1)}, V) + (1-\lambda)I(U^{(2)}, V) +$$
$$+1/2\lambda(1-\lambda)\tau_1 \cdot ||U^{(1)}(\cdot) - U^{(2)}(\cdot)||_{L_2^{r+q}}, \quad \forall U^{(k)}(\cdot) \in \mathcal{M} \quad (k = 1, 2), \quad \forall \lambda \in [0, 1].$$

And $I(U, V)$ is strongly convex in $V(\cdot)$ over $\mathcal{M}$ if there exists $\tau_2 = const > 0$ such that for every $U(\cdot) \in \mathcal{M}$

$$I(U, \lambda V^{(1)} + (1-\lambda)V^{(2)}) \leq \lambda I(U, V^{(1)}) + (1-\lambda)I(U, V^{(2)}) -$$
$$-1/2\lambda(1-\lambda)\tau_2 \cdot ||V^{(1)}(\cdot) - V^{(2)}(\cdot)||_{L_2^{r+q}}$$

for all $V^{(k)}(\cdot) \in \mathcal{M}$ $(k = 1, 2)$ and for each $\lambda \in [0, 1]$.

If functional $I(U, V)$ is strongly concave in $U(\cdot)$ over $\mathcal{M}$ and strongly convex in $V(\cdot)$ over $\mathcal{M}$ then it is said to be a strongly concave-convex over $\mathcal{M}^2$. Note, that the notion of strong concavity (convexity) of functionals can be given for the convex sets $\mathcal{M}$ only.

Assertion 2. Let the functional $I(U, V)$ from (11), (3) be strongly convex-concave over $\mathcal{M}^2$ and there exists a saddle point $(U^0(\cdot), V^0(\cdot)) \in \mathcal{M}^2$ in the game (10). Then

1. this saddle point is unique,
2. $U^0(t) = V^0(t)$ for almost all $t \in [t_*, \vartheta]$.

Доказательство. Assume that in the game (10) there exists a saddle point $(U^{(1)}(\cdot), V^{(1)}(\cdot)) \in \mathcal{M}^2$ such that $V^0(t) \neq V^{(1)}(t)$ on the zero-measure set. Then the properties of interchangeability and equivalence imply ([6, p. 40]) the couples $(U^0(\cdot), V^{(1)}(\cdot))$ and $(U^{(1)}(\cdot), V^0(\cdot))$ are the saddle points of the functional $I(U, V)$. Moreover,

$$I(U^0(\cdot), V^0(\cdot)) = I(U^0(\cdot), V^{(1)}(\cdot)) = I(U^{(1)}(\cdot), V^0(\cdot)) = I(U^{(1)}(\cdot), V^{(1)}(\cdot)).$$

In view of strong convexity of $I(U^0, V)$ in $V(\cdot)$ over $\mathcal{M}$ and convexity of $\mathcal{M}$ for the strategy $\tilde{V}(\cdot) = 1/2[V^0(\cdot) + V^{(1)}(\cdot)]$ we have the strict inequality

$$I(U^0, \tilde{V}) = I(U^0, 1/2[V^0 + V^{(1)}]) \leq 1/2I(U^0, V^0) + 1/2I(U^0, V^{(1)}) -$$
$$-1/2 \cdot 1/2 \cdot 1/2 \cdot \tau_2||V^0(\cdot) - V^{(1)}||_{L_2^{r+q}} < 1/2I(U^0, V^0) + 1/2I(U^0, V^{(1)}) =$$
$$= I(U^0, V^0)$$

contradicting (12). This contradiction implies the uniqueness of the saddle point of functional $I(U, V)$.

Now we establish the equality

$$U^0(t) = V^0(t) \quad \text{for almost all} \quad t \in [t_*, \vartheta]. \tag{13}$$

Really, combining (11) and (12) for $U^0(\cdot) = (u_1^0(\cdot), u_2^0(\cdot)) \in \mathcal{M}$ and $V^0(\cdot) = (v_1^0(\cdot), v_2^0(\cdot)) \in \mathcal{M}$ we get

$$\sum_{i \in \mathbf{N}}[J^{(i)}(u_1, v_2^0) - J^{(i)}(v_1^0, u_2)] \leq \sum_{i \in \mathbf{N}}[J^{(i)}(u_1^0, v_2^0) - J^{(i)}(v_1^0, u_2^0)] \leq$$
$$\leq \sum_{i \in \mathbf{N}}[J^{(i)}(u_1^0, v_2) - J^{(i)}(v_1, u_2^0)], \quad \forall u_1(\cdot), v_1(\cdot) \in \mathcal{U}, \quad \forall u_2(\cdot), v_2(\cdot) \in \mathcal{V}. \tag{14}$$

Let in the left part of (14) $u_1(\cdot) = v_1^0(\cdot)$ and $u_2(\cdot) = v_2^0(\cdot)$ and in the right part $v_1(\cdot) = u_1^0(\cdot)$, $v_2(\cdot) = u_2^0(\cdot)$. Then we have

$$I(V^0, V^0) = 0 \leq I(U^0, V^0) \leq I(U^0, U^0) = 0.$$

Hence at first

$$I(V^0, V^0) = I(U^0, V^0) = I(U^0, U^0) = 0.$$

Therefore (12) the couples (V^0, V^0), (U^0, V^0), (V^0, V^0) are saddle points of the functional $I(U, V)$.

At second, due to the uniqueness of such saddle point we get (13). □

Assertion 3. Under condition of Assertion 2 the couple $(u^G(\cdot), v^G(\cdot)) = (u_1^0(\cdot), u_2^0(\cdot)) \in \mathcal{U} \times \mathcal{V}$ is a Nash equilibrium situation in non-cooperative game (7), where the constants $\alpha_i = \beta_i = 1$ $(i \in \mathbf{N})$.

Доказательство. In view of Assertion 2 the left part of (14) is introduced in the form

$$\sum_{i\in\mathbf{N}}[J^{(i)}(u_1, u_2^0) - J^{(i)}(u_1^0, u_2)] \leq \sum_{i\in\mathbf{N}}[J^{(i)}(u_1^0, u_2^0) - J^{(i)}(u_1^0, u_2^0)] \tag{15}$$

$$\forall u_1(\cdot) \in \mathcal{U}, u_2(\cdot) \in \mathcal{V}.$$

Let in (15) $u_2(\cdot) = u_2^0(\cdot)$. Then we have

$$\max_{u_1(\cdot)\in\mathcal{U}} \sum_{i\in\mathbf{N}} J^{(i)}(u_1, u_2^0) = \sum_{i\in\mathbf{N}} J^{(i)}(u_1^0, u_2^0)$$

and for $u_1(\cdot) = u_1^0(\cdot)$ from (15) we get

$$\max_{u_2(\cdot)\in\mathcal{V}} [-\sum_{i\in\mathbf{N}} J^{(i)}(u_1^0, u_2)] = -\sum_{i\in\mathbf{N}} J^{(i)}(u_1^0, u_2^0).$$

Thus for $\alpha_i = \beta_i = 1$ $(i \in \mathbf{N})$ the couple $(u^G(\cdot), v^G(\cdot)) = (u_1^0(\cdot), u_2^0(\cdot)) \in \mathcal{U} \times \mathcal{V}$ is a Nash equilibrium situation in non- cooperative game (7) (this couple satisfies (6) for $\alpha_i = \beta_i = 1$ $(i \in \mathbf{N})$). □

3. Existence.

Theorem. Let in the problem (1) the functional $\sum_{i\in\mathbf{N}} J^{(i)}(u, v)$ be twice continuously differentiable and strongly concave in $u(\cdot)$ over $\mathcal{U}$ and strongly convex in $v(\cdot)$ over $\mathcal{V}$. Then the Geoffrion guaranteed solution $(u^G(\cdot), J^G) \in \mathcal{U} \times \mathbf{R}^N$ exists in this problem.

Доказательство. The strong concavity-convexity of $\sum_{i\in\mathbf{N}} J^{(i)}(u, v)$ implies the strong concavity-convexity of $\sum_{i\in\mathbf{N}} J^{(i)}(u_1, v_2)$ (where $u_1(\cdot) \in \mathcal{U}$ and $v_2(\cdot) \in \mathcal{V}$) and implies the strong convexity in $v_1(\cdot)$ over $\mathcal{U}$ and strong concavity in $u_2(\cdot)$ over $\mathcal{V}$ of the functional $-\sum_{i\in\mathbf{N}} J^{(i)}(v_1, u_2)$. Then the sum

$$\sum_{i\in\mathbf{N}} J^{(i)}(u_1, v_2) - \sum_{i\in\mathbf{N}} J^{(i)}(v_1, u_2) \tag{16}$$

is strongly concave in $U(\cdot) = (u_1(\cdot), u_2(\cdot))$ over $\mathcal{U} \times \mathcal{V}$ and strongly convex in $V(\cdot) = (v_1(\cdot), v_2(\cdot))$ over $\mathcal{U} \times \mathcal{V} = \mathcal{M}$. So the functional (16) is strongly concave in $V(\cdot)$ over $\mathcal{M}$ for every $U(\cdot) \in \mathcal{M}$ and is strongly convex in $U(\cdot)$ over $\mathcal{M}$ for every $V(\cdot) \in \mathcal{M}$

Let $\mathcal{N}_1$ be an open set from $L_2^{r+q}[t_*, \vartheta]$ such that the convex set $\mathcal{U} \times \mathcal{V} = \mathcal{M} \subset \mathcal{N}_1$. Moreover let $\mathcal{N}_2$ be some open set and $\mathcal{M} \times \mathcal{M} \subset \mathcal{N}_1^2 \subset \mathcal{N}_2$. If $\sum_{i\in\mathbf{N}} J^{(i)}(u, v)$ is twice continuously differentiable over $\mathcal{N}_1$ then the functional (16) is twice continuously differentiable over $\mathcal{N}_2$. The strong concavity-convexity and twice continuous-differentiability of (16) over $\mathcal{M} \times \mathcal{M}$ imply (see [7]) the existence of a saddle point $(U^0(\cdot), V^0(\cdot)) \in \mathcal{M}^2$. Due to Assertion 2 this saddle point is unique and $V^0(t) = U^0(t) = (u^G(\cdot), v^G(\cdot)) \in \mathcal{U} \times \mathcal{V}$ for almost all $t \in [t_*, \vartheta]$. In view of

Assertion 3 we conclude that the couple $(u^G(\cdot), v^G(\cdot)) \in \mathcal{U} \times \mathcal{V}$ is a Nash equilibrium situation in the game (7) for $\alpha_i = \beta_i = 1$ $(i \in \mathbf{N})$ and, besides, (see Assertion 1) this couple is a saddle point by Geoffrion for the problem (1).

Thus, we establish that under the conditions of the theorem there exists a saddle point by Geoffrion $(u^G(\cdot), v^G(\cdot)) \in \mathcal{U} \times \mathcal{V}$ in the problem (1) and hence (see Remark 2) there exists a Geoffrion guaranteed solution $(u^G(\cdot), J(u^G, v^G)) \in \mathcal{U} \times \mathbf{R}^N$ for this problem. □

Remark 5. From the theorem we obtain the method for constructing the Geoffrion solution $(u^G(\cdot), J(u^G, v^G)) \in \mathcal{U} \times \mathbf{R}^N$ in the problem (1):

- to introduce a dynamical system

$$\dot{y} = \phi(t, y, u_1, v_2), \quad y(t_*) = x_*,$$
$$\dot{z} = \phi(t, z, v_1, u_2), \quad z(t_*) = x_*, \tag{17}$$

and functionals

$$J^{(i)}(u_1, v_2) = \Phi^{(i)}(y(\vartheta)) + \int_{t_*}^{\vartheta} F^{(i)}(t, y(t), u_1(t), v_2(t))dt, \quad i \in \mathbf{N} = \{1, .., N\}, \tag{18}$$

$$J^{(i)}(v_1, u_2) = \Phi^{(i)}(z(\vartheta)) + \int_{t_*}^{\vartheta} F^{(i)}(t, z(t), v_1(t), u_2(t))dt, \quad i \in \mathbf{N} = \{1, .., N\}, \tag{19}$$

where $u_1(\cdot), v_1(\cdot) \in \mathcal{U}$ and $v_2(\cdot), u_2(\cdot) \in \mathcal{V}$;

- for the functional

$$\sum_{i \in \mathbf{N}} J^{(i)}(u_1, v_2) - \sum_{i \in \mathbf{N}} J^{(i)}(v_1, u_2) - \max_{U(\cdot)} \min_{V(\cdot)}$$

under the restrictions (17) and (19) to find the saddle point $(U^0(\cdot), V^0(\cdot)) \in \mathcal{M}^2$, where $U^0(\cdot) = (u_1^0(\cdot), u_2^0(\cdot)) \in \mathcal{U} \times \mathcal{V} = \mathcal{M}$, $V^0(\cdot) = (v_1^0(\cdot), v_2^0(\cdot)) \in \mathcal{M}$;

- to find the values $J^{(i)}(u_1^0, u_2^0)$ $(i \in \mathbf{N})$ and to construct the vector $J(u_1^0, u_2^0) = (J^{(1)}(u_1^0, u_2^0), .., J^{(N)}(u_1^0, u_2^0)) = J^G$.

Then the couple $(u_1^0(\cdot), u_2^0(\cdot)) = (u^G(\cdot), v^G(\cdot)) \in \mathcal{U} \times \mathcal{V}$ will be a saddle point by Geoffrion in the problem (1) and $(u^G(\cdot), J^G)$ will be the Geoffrion guaranteed solution of this problem.

Remark 6.

In order that the functional $\Phi(x(\vartheta)) + \int_{t_*}^{\vartheta} F(t, x(t), u(t), v(t))dt = \sum_{i \in \mathbf{N}} J^{(i)}(u, v)$ is strongly convex-concave over $\mathcal{M}$ the functions $F(t, x, u, v) = \sum_{i \in \mathbf{N}} F^{(i)}(t, x, u, v)$, $\Phi(x) = \sum_{i \in \mathbf{N}} \Phi^{(i)}(x)$ and the right parts of the system (2) must satisfy some restrictions. These restrictions can be found in [7].

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