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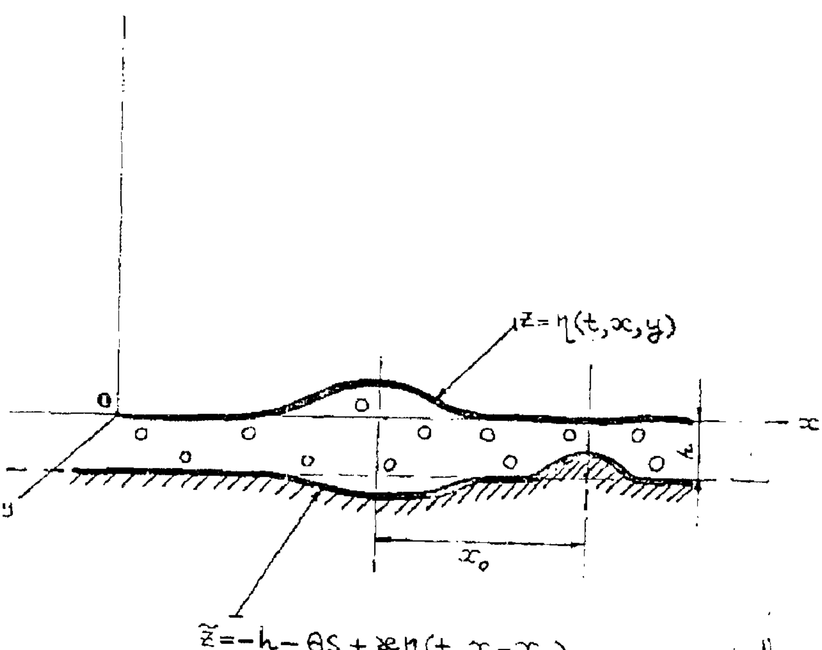
EVOLUTION OF SOLITONS

In the report we study questions of the interactions of the flow of an incompressible fluid with surfaces of bodies having visco-elastic properties, and also the possibility of reaction on the inhomogeneity of the flow by a well-defined change of shape of their surface. We solve the optimization problem of determining the shape of a surface which realizes optimum damping with respect to speed of response of isolated inhomogeneities of the fluid flow. We also consider the motion of a shallow layer of fluid over a surface with filtering layer on bottom.

We describe the property of surface by function

$$f = \theta\left(\eta - K\int_0^t \Gamma(t-\tau)\eta\, d\tau\right) - \kappa\eta(t, x - x_0, y),$$

where $\theta = G^{-1}$ is a small dimensionless parameter characterizing of elastic properties of the model, K is the coefficient of viscosity, $\Gamma(t-\tau)$ is the relaxation kernel, and $\kappa > 0$ is a dimensionless parameter characterizing the amplitude of resistance.



After transformation, which used in theory of shallow fluid we have perturbed equation KdV

$$u_t - 6uu_x + u_{xxx} = c_0\left(u_x - K\int_0^t \Gamma(t-\tau)u_x\, d\tau\right) - c_0\kappa u_x(t, x - x_0),$$

where c_0 is constant proportional of sound velocity.

Unperturbed KdV equation has a solution of the form

$$\overline{u} = 3\alpha^2 \mathrm{sech}^2 \left[\frac{1}{2}\alpha(x - \alpha^2 t)\right]$$

and the integral

$$V_0 = \frac{1}{2}\int\limits_{-\infty}^{\infty} u^2\, dx.$$

Assuming $\alpha = \alpha(t)$ we now differentiate the integral $V_0 = V_0(u)$ on the basis of the complete equation $V_0 = V_0(t)$ and $\overline{u} \in L_1(-\infty, \infty)$.

We obtain an equation for $\alpha(t)$

$$3\gamma_0 \frac{d}{dt}\alpha(t) = \frac{Kc_0}{2G}\int\limits_{-\infty}^{\infty} \mathrm{sech}^2\left[\frac{1}{2}\alpha(t)(x - \alpha^2 t)\right]\int\limits_0^t \Gamma(t-\tau)\alpha^3(\tau)\times$$

$$\times \tanh\left[\frac{1}{2}\alpha(t)(x - \alpha^2(\tau)\tau)\right]\mathrm{sech}^2\left[\frac{1}{2}\alpha(\tau)(x - \alpha^2(\tau)\tau)\right] d\tau dx +$$

$$+ \frac{1}{2}c_0 k \int\limits_{-\infty}^{\infty} \alpha^3(t)\mathrm{sech}^2\left[\frac{1}{2}\alpha(x - \alpha^2 t)\right]\times$$

$$\times\, \mathrm{sech}^2\left[\frac{1}{2}\alpha(x - x_0 - \alpha^2 t)\right]\tanh\left[\frac{1}{2}\alpha(x - x_0 - \alpha^2 t)\right] dx.$$

Assuming now $\theta \ll k$, we consider the equation, where the outstripping resistance $u_x(t, x - x_0)$ is the main factor; $\gamma_0 = \int\limits_{-\infty}^{\infty} \mathrm{sech}^4(\xi)\, d\xi = 4/3$;

$$u_t - 6uu_x + u_{xxx} = -\nu u_x(t, x - x_0), \quad \nu = \frac{c_0 k}{2}.$$

Theorem. *This equation has an asymptotic solution of the form*

$$u = 3\alpha^2 \mathrm{sech}^2\left[\frac{1}{2}\alpha(x - \alpha^2 t)\right],$$

where $\alpha = \alpha(t)$ *satisfies equation*

$$\dot{\alpha} = 4\nu p\alpha^2(3(p^2 - 1) - (p^2 + 4p + 1)\ln p)/(p - 1)^4,$$

where $p = e^{x_0\alpha}$.

We shall consider equation for $\alpha(t)$. If $0 < \alpha \ll 1$ expanding the right side of this equation in power of $\alpha(t)$ and separating out the leading term, we obtain

$$\dot{\alpha} = -\frac{2}{15}\nu x_0 \alpha^3 + o(\alpha^3).$$

We denote by $t(x_0)$ the time after which the solution — like solution dies out from initial amplitude $3\alpha_0^2$ to $(3/4)\alpha_0^2$, i.e. when the solution $\alpha = \alpha(t)$ dies out from an initial value $\alpha(0) = \alpha_0$ to $(1/2)\alpha_0$. The function $t(x_0)$ has a minimum $t_{min} = t_{min}(x_0^{min})$, $x_0^{min} = \varphi_0\alpha_0^{-1}$, where φ_0 is a constant.

The expression obtained for $x_0^{min}(\alpha_0)$ in the case $x_0 = \mathrm{const}$ implies that the outstripping resistance must be situated during the propagation of the wave at a distance from the point of maximal amplitude equal to the value of the coordinate x_0, at which the amplitude of the solution at time $t = 0$ decreases $\gamma - 1$ times, $0 < \gamma < 1$. We note that by this method it is possible to solve the problem of optimizing the outstripping speed of response with respect to shape.

The case of shallow water and under the presence of a filtering layer for $z < -h$. Then under the hypothesis of incompressibility and with filtering coefficient k_0 we obtain the equation for integral $\int\limits_{-\infty}^{\infty} u^2(t,x)\,dx$.

$$0,5 \int\limits_{-\infty}^{\infty} u^2\,dx = K \int\limits_{-\infty}^{\infty} uR\,dx,$$

where R is a perturbation in KdV equation connected with filtration. So, for $\alpha(t)$ we obtain the following equation

$$\dot{\alpha}(t) = -\gamma_1(t)\alpha, \quad \gamma_1(t) > 0.$$

Thus, the filtering layer (or the layer with similar properties) has the property of extinguishing the perturbation.

Thus, the outstripping resistance of the surface of flow, organized as a consequence of a special deformation of this surface of the body. It has been established that similar properties are possessed by the skin of dolphins and cetaceans generally due to the highly organized innervation of the tissues and the highly developed circulatory and muscular systems. All possible inhomodeneities are here suppressed including those of turbulence character, the hydrodynamic resistance is reduced in several times, and the flow thus becomes almost laminar even for high flow speed.

These physical reqularities may become a basis for the construction under artificial conditions of surfaces and bodies possessing a capacity for effective suppression of perturbations and reduction hydrodynamic resistance.

This result have published partially in the paper "On suppression of soliton-like solutions of shallow-water equations due to outstripping resistance", Soviet Math. Dokl. (1988), **37**, No. 3, 777–783 by K. V. Mal'kov and M. M. Khapaev.

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