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ON OPERATORS COLLINEAR TO THE J-ISOMETRIES

We consider linear operators acting in Kreĭn space H with an indefinite metric $[x, y] = [Jx, y]$, $J = P_+ - P_-$, $x, y \in H$, where P_+ and P_- are mutually complementary orthoprojectors. The symbol $\mathcal{L}$ below denotes an arbitrary linear manifold in H, $\mathcal{L} \neq 0$, $\mathcal{L} \subset \mathcal{D}_T$, where $\mathcal{D}_T$ is domain of the linear operator T.

Theorem. *Let operator* $(T|\mathcal{L})$ *be collinear to the* J*-isometric one, i.e.*

$$(T \mid \mathcal{L}) = \mu \cdot V,$$

where $\mu \in C$, $\mu \neq 0$, *and the operator* V *satisfies condition*

$$[Vx, Vy] = [x, y] \quad \forall x, y \in \mathcal{L}.$$

Then the three following conditions are equivalent:

1. The operator $(T \mid \mathcal{L})$ *is collinear to some uniformly* J*-nonexpansive operator, i.e.*

$$(T \mid \mathcal{L}) = \lambda \cdot U,$$

where the operator U *complies with condition*

$$[Ux, Ux] \leq [x, x] - \delta \mid\mid x \mid\mid^2$$

$$\forall x \in \mathcal{L} \quad (\delta > 0.)$$

2. The linear manifold $\mathcal{L}$ *is uniformly definite.*

3. The operator $(T|\mathcal{L})$ *is collinear to some uniformly* J*-noncontractive operator, i.e.*

$$(T \mid \mathcal{L}) = \nu \cdot W,$$

where the operator W *satisfies condition*

$$[Wx, Wx] \geq [x, x] + \tau \cdot \mid\mid x \mid\mid^2$$

$$\forall x \in \mathcal{L} \quad (\tau > 0).$$

The two following statements make more precise the values of numbers λ and ν in the main theorem.

Lemma 1. *If the linear manifold* $\mathcal{L}$ *is uniformly negative* ($[x, x] \leq \gamma_- \cdot \|x\|^2 \ \forall x \in \mathcal{L}$ $(\gamma_- < 0)$,) *then:*

1) For any $\delta > 0$ *the formula*

$$\lambda = \lambda(\delta) = \mid \mu \mid \sqrt{\frac{\gamma_-}{\gamma_- - \delta}}$$

is true.

2) For any $\tau \in (0; -\gamma_-)$ *the formula*

$$\nu = \nu(\tau) = \mid \mu \mid \sqrt{\frac{\gamma_-}{\gamma_- + \tau}}$$

is true.

Lemma 2. *If the linear manifold* $\mathcal{L}$ *is uniformly positive* ($[x, x] \geq \gamma_+ \|x\|^2 \ \forall x \in \mathcal{L}$, $(\gamma_+ > 0)$)*, then:*

1) For any $\delta \in (0; \gamma_+)$ *we have*

$$\lambda = \lambda(\delta) = \mid \mu \mid \sqrt{\frac{\gamma_+}{\gamma_+ - \delta}}.$$

2) For any $\tau > 0$ *we have*

$$\nu = \nu(\tau) = \mid \mu \mid \sqrt{\frac{\gamma_+}{\gamma_+ + \tau}}.$$

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